

Name: _____

1. A closet contains 10 pairs of shoes. If 8 shoes are randomly selected, what is the probability that there will be no complete pair? [COUNTING]

Note: in order to get full credit for this problem, use a counting argument and explain (in words) each piece of the final probability.

Ans: There are $\binom{20}{8}$ ways to select the 8 shoes from the 20 in the closet, and all of these selections are equally likely.

There are 10 pairs of shoes, and if there are no complete pairs, then at most one shoe from each pair can be selected. There are $\binom{10}{8}$ ways to choose *which* pairs contribute one shoe each, and two possibilities for picking the shoe from each of the pairs (left or right shoe). This gives a total of $\binom{10}{8} \times 2^8$ ways to choose 8 shoes that are unpaired. Thus, the probability of choosing no complete pair is:

$$\frac{\binom{10}{8} 2^8}{\binom{20}{8}}$$

2. A mother of two children is invited to a dinner for those parents who have at least one son. Given that she is invited, what is the probability that she has two sons? (Assume that boys and girls are equally likely to be born.) [CONDITIONAL PROBABILITY]

Note: in order to get credit for this problem you must specifically write down the events (more than one event!) of interest and their respective probabilities. A simple number is not sufficient.

Ans: There is a tendency to think that the answer to this problem is $1/2$ because the other child is equally likely to be a boy or a girl. But you have to pay attention to the original sample space and the conditional sample space.

Note: the original sample space (with all equally likely events) is $S = \{(b, b), (b, g), (g, b), (g, g)\}$.

$$P((b, b) | \text{at least one boy}) = \frac{P((b, b))}{P(\text{at least one boy})} = \frac{1/4}{3/4} = 1/3 \neq 1/2$$