Math 152 - Statistical Theory - Homework 10

write your name here

Due: Friday, October 23, 2020, midnight PDT

Important Note:

You should work to turn in assignments that are clear, communicative, and concise. Part of what you need to do is not print pages and pages of output. Additionally, you should remove these exact sentences and the information about HW scoring below.

Click on the *Knit to PDF* icon at the top of R Studio to run the R code and create a PDF document simultaneously. [PDF will only work if either (1) you are using R on the network, or (2) you have LaTeX installed on your computer. Lightweight LaTeX installation here: https://yihui.name/tinytex/]

> Either use the college's RStudio server (https://rstudio.pomona.edu/) or install R and R Studio on to your personal computer. See: https://research.pomona.edu/johardin/math152f20/ for resources.

Assignment

1: PodQ

Describe one thing you learned from someone in your pod this week (it could be: content, logistical help, background material, R information, etc.) 1-3 sentences.

2: 9.1.1

Let X have the exponential distribution with parameter β. Suppose that we wish to test the hypotheses $H_0 : \beta \geq 1$ versus $H_1 : \beta < 1$.

Consider the test procedure δ that rejects H_0 if $X \geq 1$.

a. Determine the power function of the test.
b. Compute the size of the test.

3: 9.1.6

Suppose that a single observation X is to be taken from the uniform distribution on the interval $[\theta - \frac{1}{2}, \theta + \frac{1}{2}]$, and suppose that the following hypotheses are to be tested:

$H_0 : \theta \leq 3,$
$H_1 : \theta \geq 4.$

Construct a test procedure δ for which the power function has the following values: $\pi(\theta|\delta) = 0$ for $\theta \leq 3$ and $\pi(\theta|\delta) = 1$ for $\theta \geq 4$.

4: 9.1.9

Assume that $X_1, \ldots, X_n$ are i.i.d. with the normal distribution that has mean μ and variance 1. Suppose that we wish to test the hypotheses:

$H_0 : \mu \geq \mu_0$,
$H_1 : \mu < \mu_0$.

Find a test statistic T such that, for every c, the test δ_c that rejects H_0 when $T \geq c$ has power function $\pi(\mu|\delta_c)$ that is decreasing in μ.

5: 9.1.13

Let X have the Poisson distribution with mean θ. Suppose that we wish to test the hypotheses:

$H_0 : \theta \leq 1.0$,
$H_1 : \theta > 1.0$.

Let δ_c be the test that rejects H_0 if $X \geq c$. Find c to make the size of δ_c as close as possible to 0.1 without being larger than 0.1.

Note: in R use `ppois()`.

6: 9.1.14

Let $X_1, \ldots, X_n$ be i.i.d. with the exponential distribution with parameter θ. Suppose that we wish to test the hypotheses:

$H_0 : \theta \geq \theta_0$,
$H_1 : \theta < \theta_0$.

Let $X = \sum_{i=1}^{n} X_i$. Let δ_c be the test that rejects H_0 if $X \geq c$.

a. Show that $\pi(\theta|\delta_c)$ is a decreasing function of θ.
b. Find c in order to make δ_c have size α_0.
c. Let $\theta_0 = 2$, $n = 1$, and $\alpha_0 = 0.1$. Find the precise form of the test δ_c and sketch its power function.

Note: $\theta * \sum(x_i) \sim$ Gamma(n,1)

7: 9.1.15

Let X have the uniform distribution on the interval $[0, \theta]$, and suppose that we wish to test the hypotheses

$H_0 : \theta \leq 1$,
$H_1 : \theta > 1$.

We shall consider test procedures of the form "reject H_0 if $X \geq c$." For each possible value x of X, find the p-value if $X = x$ is observed (that is, find the p-value as a function of x).

8: R - kissing the right way

Most people are right-handed and even the right eye is dominant for most people. Molecular biologists have suggested that late-stage human embryos tend to turn their heads to the right. German biopsychologist Onur Güntürkün conjectured that this tendency to turn to the right manifests itself in other ways as well, so he studied kissing couples to see if both people tended to lean to their right more often than to their left (and if so, how strong the tendency is). He and his researchers observed couples from age 13 to 70 in public places such as airports, train stations, beaches, and parks in the United States, Germany, and Turkey. They were careful not to include couples who were holding objects such as luggage that might have affected which direction they turned. In total, 124 kissing pairs were observed with 80 couples leaning right (Nature, 2003). [Taken from Rossman & Chance, ISCAM]

Güntürkün wanted to test the belief that the probability of kissing to the right is 3/4; he thinks it is probably less than 3/4.

a. Using the binomial distribution (not the CLT), find the rejection region for this test given a level of significance of 0.05. You can use trial and error or the inverse function for the binomial to come up with your test.

```
# for X ~ Bin(size, prob)
size=124
prob=.75
q=2
p=.1
pbinom(q, size, prob)     # gives P(X < = q)
```

```
## [1] 1.525648e-70
```

```
dbinom(q, size, prob)     # gives P(X=q)
```

```
## [1] 1.517401e-70
```

```
qbinom(p, size, prob)     # gives the cutoff for a given probability, p
```

```
## [1] 87
```

b. What is the size of your test?

c. Calculate and plot the power function over all possible values of θ. Do you think your test seems particularly powerful? Explain. (Note: you'll need to change the code below to say `eval = TRUE` and also actually write out the power function.)

```
all.theta <-  seq(0,1,.001)
all.power <-  #somefunction of all.theta and n (size)#
plot(all.theta, all.power, xlab="possible theta values", ylab="power function")
```

d. Given the data (80 couples kissed right), what is the p-value of the test?