Math 152 - Statistical Theory - Homework 11

write your name here

Due: Tuesday, November 10, 2020, midnight PDT

Important Note:

You should work to turn in assignments that are clear, communicative, and concise. Part of what you need to do is not print pages and pages of output. Additionally, you should remove these exact sentences and the information about HW scoring below.

Click on the *Knit to PDF* icon at the top of R Studio to run the R code and create a PDF document simultaneously. [PDF will only work if either (1) you are using R on the network, or (2) you have LaTeX installed on your computer. Lightweight LaTeX installation here: https://yihui.name/tinytex/]

> Either use the college's RStudio server (https://rstudio.pomona.edu/) or install R and R Studio on to your personal computer. See: https://research.pomona.edu/johardin/math152f20/ for resources.

Assignment

1: PodQ

Describe one thing you learned from someone in your pod this week (it could be: content, logistical help, background material, R information, etc.) 1-3 sentences.

2: 9.2.2

Consider two pdfs $f_0(x)$ and $f_1(x)$ that are defined as follows:

$$f_0(x) = \begin{cases} 1 & \text{for } 0 \le x \le 1, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$f_1(x) = \begin{cases} 2x & \text{for } 0 \le x \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

Suppose that a single observation X is taken from a distribution for which the pdf $f(x)$ is either $f_0(x)$ or $f_1(x)$, and the following simple hypotheses are to be tested:

$H_0 : f(x) = f_0(x),$
$H_1 : f(x) = f_1(x).$

a. Describe a test procedure for which the value of $\alpha(\delta) + 2\beta(\delta)$ is a minimum.
b. Determine the minimum value of $\alpha(\delta) + 2\beta(\delta)$ attained by that test procedure.

Convince yourself that the theorems in 9.2 hold.

3: 9.2.4

Consider again the conditions of Exercise 9.2.2, but suppose now that it is desired to find a test procedure for which $\alpha(\delta) \leq 0.1$ and $\beta(\delta)$ is a minimum.

a. Describe the test procedure.
b. Determine the minimum value of $\beta(\delta)$ attained by the test procedure.

Convince yourself that the theorems in 9.2 hold.

4: 9.2.6

Suppose that $X_1, \ldots, X_n$ form a random sample from the Bernoulli distribution with unknown parameter p. Let p_0 and p_1 be specified values such that $0 < p_1 < p_0 < 1$, and suppose that it is desired to test the following simple hypotheses:

$H_0 : p = p_0,$
$H_1 : p = p_1.$

a. Show that a test procedure for which $\alpha(\delta) + \beta(\delta)$ is a minimum rejects H_0 when $\overline{X} < c$.

b. Find the value of the constant c.

5: 9.2.7

Suppose that $X_1, \ldots, X_n$ form a random sample from the normal distribution with known mean μ and unknown variance σ^2, and the following simple hypotheses are to be tested:

$H_0 : \sigma^2 = 2,$
$H_1 : \sigma^2 = 3.$

a. Show that among all test procedures for which $\alpha(\delta) \leq 0.05$, the value of $\beta(\delta)$ is minimized by a test procedure that rejects H_0 when $\sum_{i=1}^{n}(X_i - \mu)^2 > c$.

b. For $n = 8$, find the value of the constant c that appears in part a.

6: 9.2.8

Suppose that a single observation X is taken from the uniform distribution on the interval $[0, \theta]$, where the value of θ is unknown, and the following simple hypotheses are to be tested:

$H_0 : \theta = 1,$
$H_1 : \theta = 2.$

a. Show that there exists a test procedure for which $\alpha(\delta) = 0$ and $\beta(\delta) < 1$.
b. Among all test procedures for which $\alpha(\delta) = 0$, find the one for which $\beta(\delta)$ is a minimum.

7: R - inflated errors

An unethical experimenter desires to test the following hypotheses:

$H_0 : \theta = \theta_0$
$H_1 : \theta \neq \theta_0$

She draws a random sample $X_1, X_2, \ldots, X_n$ from a distribution with the pdf $f(x|\theta)$, and carries out a test of size α_0. If this test does not reject H_0, she discards the sample, draws a new independent random sample of n observations, and repeats the test based on the new sample. She continues drawing new independent samples in this way until she obtains a sample for which H_0 is rejected.

a. What is the overall size of this testing procedure?

b. If H_0 is true, what is the expected number of samples (each with the same number of observations) that the experimenter will have to draw until she rejects H_0? (Hint: Let X = number of samples until rejecting H_0. What is the distribution of X? Is that distribution on the distribution sheet?)
c. Do a simulation in R to corroborate your answer to b. Assume the X_i are normally distributed with mean μ unknown and variance 1. Use $\alpha_0 = 0.05$. Figure out a way to display your results so that they form part of a convincing argument. (If you tell me what you want to display, I'm happy to tell you the R code to display it.)