Math 152 - Statistical Theory - Homework 12

write your name here

Due: Wednesday, November 25, 2020, midnight PDT

Important Note:

You should work to turn in assignments that are clear, communicative, and concise. Part of what you need to do is not print pages and pages of output. Additionally, you should remove these exact sentences and the information about HW scoring below.

Click on the *Knit to PDF* icon at the top of R Studio to run the R code and create a PDF document simultaneously. [PDF will only work if either (1) you are using R on the network, or (2) you have LaTeX installed on your computer. Lightweight LaTeX installation here: https://yihui.name/tinytex/]

> Either use the college's RStudio server (https://rstudio.pomona.edu/) or install R and R Studio on to your personal computer. See: https://research.pomona.edu/johardin/math152f20/ for resources.

Assignment

1: PodQ

Describe one thing you learned from someone in your pod this week (it could be: content, logistical help, background material, R information, etc.) 1-3 sentences.

2. 9.3.3

Suppose that $X_1, \ldots, X_n$ form a random sample from the gamma distribution with parameters α and β. Assume that α is unknown ($\alpha > 0$) and that β is known. Show that the joint pdf of $X_1, \ldots, X_n$ has a monotone likelihood ratio in the statistic $\prod_{i=1}^{n} X_i$.

3. 9.3.6

Suppose that $X_1, \ldots, X_n$ form a random sample from the uniform distribution on the interval $[0, \theta]$. Show that the joint pdf of $X_1, \ldots, X_n$ has a monotone likelihood ratio in the statistic $\max\{X_1, \ldots, X_n\}$.

4. 9.5.12

Suppose that a random sample $X_1, \ldots, X_n$ is to be taken from the normal distribution with unknown mean μ and unknown variance σ^2, and the following hypotheses are to be tested:
$H_0 : \mu \leq 3$,
$H_1 : \mu > 3$.

Suppose also that the sample size n is 17, and it is found from the observed values in the sample that $\overline{X} = 3.2$ and $(1/n) \sum_{i=1}^{n} (X_i - \overline{X})^2 = 0.09$. Calculate the value of the statistic U, and find the corresponding p-value.

5. 9.5.13

Consider again the conditions of Exercise 12, but suppose now that the sample size n is 170, and it is again found from the observed values in the sample that $\overline{X} = 3.2$ and $(1/n) \sum_{i=1}^{n} (X_i - \overline{X})^2 = 0.09$. Calculate the value of the statistic U and find the corresponding p-value.

6. 9.5.17

Prove that the likelihood ratio test for hypotheses (9.5.7) is the two-sided t test that rejects H0 if $|U| \geq c$, where U is defined in Eq. (8.5.1). The argument is slightly simpler than, but very similar to, the one given in the text for the one-sided case.

7. 10.1.9

The 50 values in Table 10.5 are intended to be a random sample from the standard normal distribution.

    a. Carry out a χ^2 test of goodness-of-fit by dividing the real line into five intervals, each of which has probability 0.2 under the standard normal distribution.

    b. Carry out a χ^2 test of goodness-of-fit by dividing the real line into 10 intervals, each of which has probability 0.1 under the standard normal distribution.

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data10.1.9 <- c(-1.28, -1.22, -0.45, -0.35, 0.72, -0.32, -0.80, -1.66,
                1.39, 0.38, -1.38, -1.26, 0.49, -0.14, -0.85, 2.33, -0.34,
                -1.96, -0.64, -1.32, -1.14, 0.64, 3.44, -1.67, 0.85, 0.41,
                -0.01, 0.67, -1.13, -0.41, -0.49, 0.36, -1.24, -0.04, -0.11,
                1.05, 0.04, 0.76, 0.61, -2.04, 0.35, 2.82, -0.46, -0.63,
                -1.61, 0.64, 0.56, -0.11, 0.13, -1.81)
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No R question