Math 152 - Statistical Theory - Homework 2

write your name here

Due: Friday, September 4, midnight PDT

Important Note:

You should work to turn in assignments that are clear, communicative, and concise. Part of what you need to do is not print pages and pages of output. Additionally, you should remove these exact sentences and the information about HW scoring below.

Click on the *Knit to PDF* icon at the top of R Studio to run the R code and create a PDF document simultaneously. [PDF will only work if either (1) you are using R on the network, or (2) you have LaTeX installed on your computer. Lightweight LaTeX installation here: https://yihui.name/tinytex/]

> Either use the college's RStudio server (https://rstudio.pomona.edu/) or install R and R Studio on to your personal computer. See: https://research.pomona.edu/johardin/math152f20/ for resources.

Assignment

Book problems

- Feel free to do the book problems with a pencil or in LaTeX (RMarkdown supports writing mathematics using LaTeX).
- If you use a pencil, just append your pencil pdf to the RMarkdown created when you knit your R code.
- If you have the 3rd edition of the book, the problems will be the same unless they don't exist – that is, the 4th edition *added* problems but didn't change the order of them. Ask me if you want to see the 4th edition problems.

1: PodQ

Describe one thing you learned from someone in your pod this week (it could be: content, logistical help, background material, R information, etc.) 1-3 sentences.

2: 7.2.6

Suppose that the proportion θ of defective items in a large manufactured lot is unknown, and the prior distribution of θ is the uniform distribution on the interval [0, 1]. When eight items are selected at random from the lot, it is found that exactly three of them are defective. Determine the posterior distribution of θ.

3: 7.2.9

Consider again the problem described in Exercise 6, and assume the same prior distribution of θ. Suppose now, however, that instead of selecting a random sample of eight items from the lot, we perform the following experiment: Items from the lot are selected at random one by one until exactly three defectives have been found. If we find that we must select a total of eight items in this experiment, what is the posterior distribution of θ at the end of the experiment?

4: 7.2.10

Suppose that a single observation X is to be taken from the uniform distribution on the interval [θ - 1/2, θ + 1/2], the value of θ is unknown, and the prior distribution of θ is the uniform distribution on the interval [10, 20]. If the observed value of X is 12, what is the posterior distribution of θ?

5: 7.2.11

Consider again the conditions of Exercise 10, and assume the same prior distribution of θ. Suppose now, however, that six observations are selected at random from the uniform distribution on the interval [θ - 1/2, θ + 1/2], and their values are 11.0, 11.5, 11.7, 11.1, 11.4, and 10.9. Determine the posterior distribution of θ.

6: 7.3.7

Suppose that the heights of the individuals in a certain population have a normal distribution for which the value of the mean θ is unknown and the standard deviation is 2 inches. Suppose also that the prior distribution of θ is a normal distribution for which the mean is 68 inches and the standard deviation is 1 inch. If 10 people are selected at random from the population, and their average height is found to be 69.5 inches, what is the posterior distribution of θ?

7: 7.3.8

Consider again the problem described in Exercise 7.
a. Which interval 1-inch long had the highest prior probability of containing the value of θ?
b. Which interval 1-inch long has the highest posterior probability of containing the value of θ?
c. Find the values of the probabilities in parts (a) and (b).

8: 7.3.9

Suppose that a random sample of 20 observations is taken from a normal distribution for which the value of the mean θ is unknown and the variance is 1. After the sample values have been observed, it is found that $\bar{x} = 10$, and that the posterior distribution of θ is a normal distribution for which the mean is 8 and the variance is 1/25.What was the prior distribution of θ?

9: 7.3.11

Suppose that a random sample of 100 observations is to be taken from a normal distribution for which the value of the mean θ is unknown and the standard deviation is 2, and the prior distribution of θ is a normal distribution. Show that no matter how large the standard deviation of the prior distribution is, the standard deviation of the posterior distribution will be less than 1/5.

10: R - beta-binomial family

Consider the beta-binomal family (i.e., beta prior, binomial likelihood (with parameter theta), beta posterior). That is, the parameter of interest is theta, and both the prior and posterior distributions of theta are from the beta family.

(a) Write down the posterior distribution of theta given the data as a function of prior α, prior β, n, and $\hat{p}$ = **proportion of successes**.

(b) How does the posterior expected value of theta change as a function of each of the values above?

(c) Using simulations, histograms, and means, **discuss the role of sample size** when using a prior and Bayesian inference. For the discussion:

i. give posterior histogram and sample means for the following combinations (12 histograms):

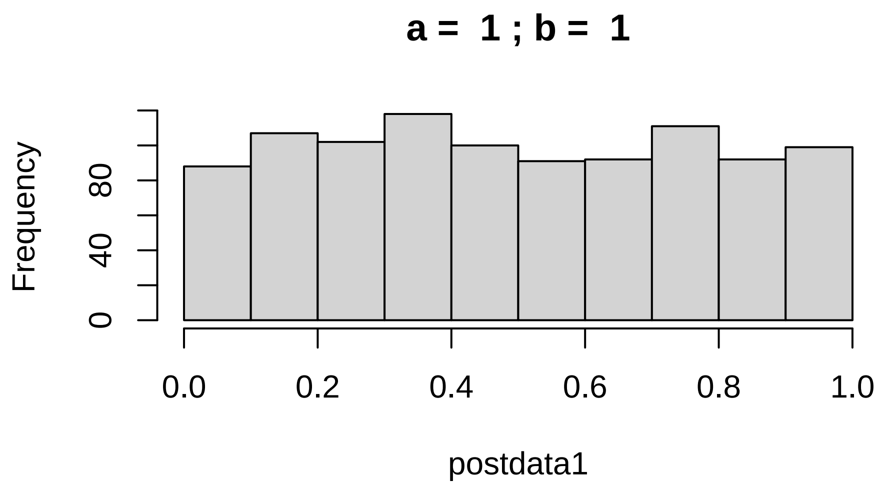
- (α, β): (4,4); (4,10)

- $\hat{p}$: 0.2, 0.5
- n: 10, 100, 1000

ii. Using your histograms and means above, discuss the role of sample size in determining the posterior distribution of the parameter.

Some R code that might be helpful:

```
a1 = 1 # you need to change this
b1 = 1 # you need to change this
postdata1 = rbeta(1000, a1, b1)
hist(postdata1, main = paste("a = ", a1, "; b = ", b1))
```




```
mean(postdata1)
```

```
## [1] 0.4983015
```

```
sd(postdata1)
```

```
## [1] 0.2864126
```