Math 152 - Statistical Theory - Homework 4

write your name here

Due: 9/28/2018

Book problems:

7.5 – 2, 3, 5, 6, 11

R problem

Example 7.6.5 in the text looks at the MLE for the center of of a Cauchy distribution. The Cauchy distribution is interesting because the tails decay at a rate of $1/x^2$, so that when you try to take the expected value, you end up integrating something that looks like $1/x$ over the real line. Hence, the expected value does not exist. Thus, method of moments estimators are of no use. The MLE is still useful, though not easy to find. As stated in the text, the likelihood is proportional to

$$\prod_{i=1}^{n}[1+(x_i-\theta)^2]^{-1}$$

(a) Compute the first and second derivative of the log likelihood.

(b) Consider trying to find the root of a function f(x). Suppose your current guess is some value x_0. We might approximate the function by the tangent line (first order Taylor approximation) at x0 and take our next guess as the root of that line. Use the Taylor expansion to find the next guess of x_1 as

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Continually updating our guesses via this method is known as Newton's Method or the Newton-Raphson Method.

Using your work from (a), find x_1 as a function of x_0.

(c) Generate 50 observations from a Cauchy distribution centered at $\theta = 10$. Based on these 50 observations, use parts (a) and (b) to estimate θ with a maximum likelihood approach. Remember, we're trying to maximize the likelihood, so the function that we are trying to find the root of is derivative of the log-likelihood. The R code might look something like this:

```
x=rt(50,1)+10  # 50 random Cauchy variables centered at 10
vect.theta = c()  # placeholder
theta.guess=((pick a starting value, you might try different ones))  # just a number


for (i in 1:10) {  # play around with how many times you loop through.  10 is likely too small.

    f1=((compute first derivative of log-likelihood evaluated at theta.guess))
    f2=((compute second derivative at theta.guess))

# f1 and f2 need to be written as R functions of theta.guess

    theta.guess=theta.guess - f1/f2
    print(theta.guess)
    vect.theta = c(vect.theta, theta.guess) # keeping track of all your guesses
```

```
}

plot(vect.theta)
```

where the actual formulas for f1 and f2 will come from part (a). Everything in double parentheses needs to be replaced by proper R syntax. The rest of it will run in R. In case you are curious, **sum()** is the R function for summing a vector.

Include plot of your guesses and your final value for the MLE of θ.