Bootstrap CIs

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There are many built in functions in R (and Python, Matlab, Stata, etc. for that matter) which will bootstrap a dataset and create any of a number of standard bootstrap intervals. However, this file will bootstrap using first principles in order to see the inner workings of the bootstrap process.

Example: heroin survival time

- The data include the amount of time that the subjects stayed in the facility until treatment was terminated (column 4).
- For about 37% of the subjects, the study ended while they were still in clinic (status=0).
- Their survival time has been truncated. For this reason we might not want to estimate the mean survival time, but rather some other measure of typical survival time. Below we explore using the median as well as the 25% trimmed mean. (From ISCAM Chance & Rossman, Investigation 4.5.3)

```r
heroin <- readr::read_table2("http://www.rossmanchance.com/iscam2/data/heroin.txt")
names(heroin)
```

```r
## [1] "id"   "clinic" "status" "times" "prison" "dose"
```

```r
head(heroin)
```

```r
## # A tibble: 6 x 6
##   id clinic status times prison dose
##  <dbl> <dbl> <dbl>  <dbl> <dbl> <dbl>
## 1     1     1      1      428    0    50
## 2     2     1      1      275    1    55
## 3     3     1      1      262    0    55
## 4     4     1      1      183    0    30
## 5     5     1      1      259    1    65
## 6     6     1      1      714    0    55
```

```r
obs.stat <- heroin %>%
  summarize(tmeantime = mean(times, trim=0.25)) %>% pull()
obs.stat
```

```r
## [1] 378.3
```
Bootstrapping the data

```r
set.seed(4747)
heroin.bs <- heroin %>% sample_frac(size=1, replace=TRUE)

heroin.bs %>%
  summarize(tmeantime = mean(times, trim=0.25)) %>% pull()

## [1] 372.2583
```

Creating a sampling distribution for the trimmed mean

```r
bs.test.stat <- c()  # will eventually be B long, check after you run everything!
bs.sd.test.stat <- c()  # will eventually be B long, check after you run everything!

B <- 500
M <- 100
set.seed(4747)

for(b in 1:B){
  heroin.bs <- heroin %>% sample_frac(size=1, replace=TRUE)
  bs.test.stat <- c(bs.test.stat,
                   heroin.bs %>%
                     summarize(tmeantime = mean(times, trim = 0.25)) %>% pull())

  bsbs.test.stat <- c()  # refresh the rs test statistics

  for(m in 1:M){
    heroin.bsbs <- heroin.bs %>% sample_frac(size=1, replace=TRUE)
    bsbs.test.stat <- c(bsbs.test.stat,
                        heroin.bsbs %>%
                          summarize(tmeantime = mean(times, trim = 0.25)) %>% pull())
  }

  bs.sd.test.stat <- c(bs.sd.test.stat, sd(bsbs.test.stat))
}
```
What do the data distributions look like?

original sample

one bootstrap sample

a bootstrap sample of the one bootstrap sample
What do the sampling distributions look like?

**dist of trimmed mean**

```
trimmed.mean = 379.81 ; SE = 23.04
```

What is the distribution of the SE of the statistic?

**dist of SE of trimmed means**

```
average SE = 22.31
```

What is the distribution of the T statistics?

**dist of T statistics of trimmed means**

```
average T = 0.07
```
95% normal CI with BS SE

\[
\text{obs.stat} + qnorm(c(.025,.975)) \times \text{sd(bs.test.stat)}
\]

## [1] 333.1345 423.4655

95% Bootstrap-t CI

Note that the t-value is needed (which requires a different SE for each bootstrap sample).

\[
\text{bs.t.hat} = (\text{bs.test.stat} - \text{obs.stat}) / \text{bs.sd.test.stat}
\]

\[
\text{bs.t.hat.95} = \text{quantile(bs.t.hat, c(.025,.975))}
\]

\[
\text{obs.stat} + \text{bs.t.hat.95} \times \text{sd(bs.test.stat)}
\]

## 2.5% 97.5%
## 335.4009 431.3040

95% Percentile CI

\[
\text{quantile(bs.test.stat, c(.025,.975))}
\]

## 2.5% 97.5%
## 339.3323 431.0694