Math 152 – Fall 2008
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warm-up # 10

Name: ________________________________

Let $X_1, X_2, \ldots, X_n$ be distributed independently and identically with mean γ and variance ν. Find:

1. $E[\overline{X}]$
2. $Var[\overline{X}]$
3. $E\left[\frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2\right]$

Solution:

1. $E[\overline{X}] = (1/n)\sum E[X_i] = (1/n)\sum \gamma = \gamma$
2. $Var[\overline{X}] = (1/n^2)\sum Var[X_i] = (1/n^2)n \cdot \nu = \nu/n$
3. In your notes:

$$
\begin{aligned}
E\left[\frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2\right] &= \frac{1}{n}E[\sum(X_i - \gamma + \gamma - \overline{X})^2] \\
&= \frac{1}{n}E[\sum(X_i - \gamma)^2 + 2\sum(X_i - \gamma)(\gamma - \overline{X}) + n(\gamma - \overline{X})^2] \\
&= \frac{1}{n}\{E[\sum(X_i - \gamma)^2] - 2E[(\overline{X} - \gamma)\sum(X_i - \gamma)] + nE[(\overline{X} - \gamma)^2]\} \\
&= \frac{1}{n}\{\sum E[(X_i - \gamma)^2] - 2nE[(\overline{X} - \gamma)^2] + n\frac{\nu}{n}\} \\
&= \frac{1}{n}\{n\nu - 2n\frac{\nu}{n} + \nu\} \\
&= \frac{n-1}{n}\nu \Rightarrow \text{Biased!}
\end{aligned}
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