

Name: \_\_\_\_\_

Assume weights of cereal in 10oz boxes are normally distributed,  $N(\mu, \sigma^2)$ , both unknown. To test whether or not the box label is accurate, we set up our hypotheses:

$$H_0 : \mu = 10\text{oz}$$

$$H_1 : \mu \neq 10\text{oz}$$

- Find the critical region for this test with a size of 0.05.
- Also, given a sample of size 16 cereal boxes with a mean weight of 10.4oz and a standard deviation of 0.85oz, find the p-value for the data.

### Solution

What does the form of our critical region look like? (reject if less than a certain amount below 10oz, or bigger than a certain amount above 10oz, draw a number line with shaded rejection region) We will reject  $H_0$  if  $\bar{X}$  is too big or too small.

If  $|\bar{X} - 10|$  is too big, then  $\frac{|\bar{X}-10|}{s/\sqrt{n}}$  will also be too big.

$$C = \left\{ \frac{|\bar{X} - 10|}{s/\sqrt{n}} > c \right\}$$
$$\delta : \left\{ \text{reject } H_0 \text{ if } T \in C \text{ where } T = \frac{|\bar{X} - 10|}{s/\sqrt{n}} \right\}$$

$$\begin{aligned} \alpha(\delta) &= \sup_{\mu \in \Omega_0} \pi(\mu|\delta) \\ &= P\left(\frac{|\bar{X} - 10|}{s/\sqrt{n}} > c \mid \mu = 10\right) \\ &= P(-c < \frac{|\bar{X} - 10|}{s/\sqrt{n}} < c) \\ &= P(-c < t_{15} < c) = 0.05 \\ c &= 2.131 \end{aligned}$$

Note: we also reject  $H_0$  if  $\bar{X} \notin 10 \pm 0.453$ . (More later on computing the power  $\pi(\mu|\delta)$ ,  $\mu \in \Omega_1$ .)

What is the p-value given our data?

$$\begin{aligned}\text{p-value} &= P(T > \hat{T}) \\ &= P\left(\frac{|\bar{X} - 10|}{s/\sqrt{16}} > \frac{|10.4 - 10|}{0.85/\sqrt{16}}\right) \\ &= 2P(t_{15} > 1.88) \\ 1.753 &\leq 1.88 \leq 2.131 \\ 2 \cdot 0.025 &\leq \text{p-value} \leq 2 \cdot 0.05 \\ 0.05 &\leq \text{p-value} \leq 0.10\end{aligned}$$

There is moderate evidence (not strong) to say that cereal boxes weigh, on average, something other than 10oz.