Math 152 – Fall 2008
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warm-up # 13

Name: ________________________________

Assume weights of cereal in 10oz boxes are normally distributed, $N(\mu, \sigma^2)$, both unknown. To test whether or not the box label is accurate, we set up our hypotheses:

$$H_0 : \mu = 10\text{oz}$$
$$H_1 : \mu \neq 10\text{oz}$$

- Find the critical region for this test.
- Also, given a sample of size 16 cereal boxes with a mean weight of 10.4oz and a standard deviation of 0.85oz, find the p-value for the data.

Solution
What does the form of our critical region look like? (reject if less than a certain amount below 10oz, or bigger than a certain amount above 10oz, draw a number line with shaded rejection region) We will reject H_0 if $\overline{X}$ is too big or too small.

If $|\overline{X} - 10|$ is too big, then $\frac{|\overline{X}-10|}{s/\sqrt{n}}$ will also be too big.

$$C = \left\{ \frac{|\overline{X} - 10|}{s/\sqrt{n}} > c \right\}$$
$$\delta : \quad \left\{ \text{reject } H_0 \text{ if } T \in C \text{ where } T = \frac{|\overline{X} - 10|}{s/\sqrt{n}} \right\}$$

$$\begin{aligned}
\alpha(\delta) &= \sup_{\mu \in \Omega_0} \pi(\mu|\delta) \\
&= P\left(\frac{|\overline{X} - 10|}{s/\sqrt{n}} > c|\mu = 10 \right) \\
&= P(-c < \frac{|\overline{X} - 10|}{s/\sqrt{n}} < c) \\
&= P(-c < t_{15} < c) = 0.05 \\
c &= 2.131
\end{aligned}$$

Note: we also reject H_0 if $\overline{X} \notin 10 \pm 0.453$. (More later on computing the power $\pi(\mu|\delta)$, $\mu \in \Omega_1$.)

What is the p-value given our data?

$$\begin{aligned}
\text{p-value} &= P(T > \hat{T}) \\
&= P\left(\frac{|\overline{X} - 10|}{s/\sqrt{16}} > \frac{|10.4 - 10|}{0.85/\sqrt{16}}\right) \\
&= 2P(t_{15} > 1.88) \\
1.753 \leq\ & 1.88 \leq 2.131 \\
2 \cdot 0.025 \leq\ & \text{p-value} \leq 2 \cdot 0.05 \\
0.05 \leq\ & \text{p-value} \leq 0.10
\end{aligned}$$

There is moderate evidence (not strong) to say that cereal boxes weigh, on average, something other than 10oz.