

Name: \_\_\_\_\_

Let's say we have 2 batches of paint one of which is quick-dry. The paint is unlabeled, and we forgot which was which! We paint 5 boards from batch 1 and record the drying time. We think batch 1 is quick dry. We also believe that the drying times are normally distributed with a standard deviation of 5 min.

$$H_0 : \theta = 25 \text{ min}$$

$$H_1 : \theta = 10 \text{ min}$$

Find  $\delta^*$  that minimizes  $\beta(\delta^*)$  subject to  $\alpha(\delta^*) < .05$ .

**Solution**

$$\delta^* : \{\text{reject } H_0 \text{ if } \bar{x} < 17.5 - \frac{5}{3n} \ln(k)\}$$

$$P(\bar{X} < 17.5 - \frac{5}{3n} \ln(k) | \theta = 25) = 0.05$$

$$P(Z < \frac{17.5 - 5/3n \ln(k) - 25}{5/\sqrt{n}}) = 0.05$$

$$\frac{17.5 - 5/3n \ln(k) - 25}{5/\sqrt{n}} = -1.68$$

$$\ln(k) = -11.23$$

$$\delta^* : \{\text{reject } H_0 \text{ if } \bar{x} < 21.24\}$$

$$\text{note: } P(\bar{X} > 21.24 | \theta = 10) = 0$$