Math 152 – Fall 2008
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warm-up # 16

Name: ________________________________

Consider a normal distribution, $N(\mu, \sigma^2)$, both parameters unknown. And suppose we wish to test the following hypotheses:

$$H_0 : \mu \leq \mu_0$$
$$H_1 : \mu > \mu_0$$

Suppose that it is possible to observe only a single value of X from this distribution, but that an independent random sample of n observations $Y_1, Y_2, \ldots, Y_n$ is available from another normal distribution for which the variance is also σ^2, but the mean is zero.

Show how to carry out a test (with size α_0) of the hypotheses H_0 and H_1 based on the t-distribution with n degrees of freedom.

Solution:
As to before, we let $U = \frac{X-\mu_0}{s_Y}$, but s_Y is based on the sample of Y_i. That is, $s_Y^2 = (1/n)\Sigma Y_i^2$. We know, if μ_0 is the true value for the mean:

$$\begin{aligned}
\frac{X-\mu_0}{\sigma} &\sim N(0,1) \\
\frac{\Sigma Y_i^2}{\sigma^2} &\sim \chi^2_n \\
\frac{X-\mu_0}{s_Y} &\sim t_n \\
\delta : \{\text{reject } H_0 &\text{ if } \quad U > c\} \\
P(U > c|\mu_0) &= P(t_n > c) = \alpha_0 \\
c &= t_{n,1-\alpha_0}
\end{aligned}$$

Will be a size α_0 test of the above hypotheses.