Math 152 – Fall 2008
Jo Hardin
warm-up # 1

Name: ________________________________

You are provided the marginal distribution of X and the conditional distribution of Y given X. Your task is to find their joint distribution and the marginal distribution of Y.

$$\begin{aligned} f_X(x) &= e^{-x} \qquad 0 < x < \infty \\ f_{Y|X=x}(Y|X=x) &= e^{-(y-x)} \qquad 0 < x < y < \infty \end{aligned}$$

1. Find the joint distribution of X and Y, $f_{X,Y}(x,y)$.
2. Find the marginal distribution of Y, $f_Y(y)$.
3. Why doesn't $f_{X,Y}(x,y) = f_X(x)f_Y(y)$?

Solution:

1.

$$\begin{aligned} f_{X,Y}(x,y) &= f_{Y|X=x}(y|X=x)f_X(x) \\ &= e^{-(y-x)}e^{-x} \quad 0 < x < y < \infty \\ &\quad \text{how does the product of the inequality work???} \\ &= e^{-y} \quad 0 < x < y < \infty \end{aligned}$$

2.

$$\begin{aligned} f_Y(y) &= \int_{-\infty}^{\infty} f(x,y)dx \\ &= \int_0^y e^{-y}dx \\ &= e^{-y} \cdot x \Big|_0^y \\ &= e^{-y}(y-0) \\ &= ye^{-y} \qquad 0 < y < \infty \end{aligned}$$

3. Because X and Y are not independent random variables.