

Name: _____

You are provided the marginal distribution of X and the conditional distribution of Y given X . Your task is to find their joint distribution and the marginal distribution of Y .

$$\begin{aligned}f_X(x) &= e^{-x} & 0 < x < \infty \\f_{Y|X=x}(y|X=x) &= e^{-(y-x)} & 0 < x < y < \infty\end{aligned}$$

1. Find the joint distribution of X and Y , $f_{X,Y}(x, y)$.
2. Find the marginal distribution of Y , $f_Y(y)$.
3. Why doesn't $f_{X,Y}(x, y) = f_X(x)f_Y(y)$?

Solution:

1.

$$\begin{aligned}f_{X,Y}(x, y) &= f_{Y|X=x}(y|X=x)f_X(x) \\&= e^{-(y-x)}e^{-x} & 0 < x < y < \infty \\&\quad \text{how does the product of the inequality work???} \\&= e^{-y} & 0 < x < y < \infty\end{aligned}$$

2.

$$\begin{aligned}f_Y(y) &= \int_{-\infty}^{\infty} f(x, y)dx \\&= \int_0^y e^{-y}dx \\&= e^{-y} \cdot x \Big|_0^y \\&= e^{-y}(y - 0) \\&= ye^{-y} & 0 < y < \infty\end{aligned}$$

3. Because X and Y are not independent random variables.