Math 152 – Fall 2008
Professor Hardin
warm-up # 20

Name: ________________________

It is proposed to fit the Poisson distribution to the following data:

x	0	1	2	3	$3 < x$
Frequency	20	40	16	18	6

Use a χ^2 goodness-of-fit test to test the Poisson distributional assumption. (Hint: in computing the mean, treat $3 < x$ as $x = 4$. Additionally, use $k - 2$ degrees of freedom because we have to estimate λ.)

Solution

$$\begin{aligned} H_0 &: \quad X \sim \text{ Poisson}(\lambda) \\ H_1 &: \quad X \text{ not } \sim \text{ Poisson}(\lambda) \end{aligned}$$

$$\begin{aligned} \hat{\lambda} &= \overline{x} = \frac{0 \cdot 20 + 1 \cdot 40 + 2 \cdot 16 + 3 \cdot 18 + 4 \cdot 6}{20 + 40 + 16 + 18 + 6} = 1.5 \\ P(X = 0|\lambda = 1.5) &= 0.2231 \\ P(X = 1|\lambda = 1.5) &= 0.3347 \\ P(X = 2|\lambda = 1.5) &= 0.2510 \\ P(X = 3|\lambda = 1.5) &= 0.1255 \\ P(X > 3|\lambda = 1.5) &= 0.0657 \end{aligned}$$

$$\begin{aligned} \chi^2 &= \sum_{i=1}^{5} \frac{(N_i - np_i^0)^2}{np_i^0} \\ &= \frac{(20 - 22.31)^2}{22.31} + \frac{(40 - 33.47)^2}{33.47} + \frac{(16 - 25.1)^2}{25.1} + \frac{(18 - 12.55)^2}{12.55} + \frac{(6 - 6.57)^2}{6.57} \\ &= 7.23 \\ P(\chi^2_{(5-2)} > 7.23) &> 0.05 \end{aligned}$$

Because our p-value is bigger than 0.1, we cannot reject the null hypothesis. We have no evidence against the data having a Poisson distribution.