Name: _____

Suppose that the number of minutes a person must wait for a bus each morning has a uniform distribution on the interval $[0, \theta]$, where the value of the endpoint θ is unknown. Suppose also that the prior pdf of θ is:

$$\xi(\theta) = \begin{cases} \frac{192}{\theta^4} & \theta \geq 4 \\ 0 & \text{else} \end{cases}$$

If the observed waiting times on three successive mornings are 5, 3, and 8 minutes, what is the posterior pdf of θ given the data?

Solution:

$$\begin{aligned} X_1, X_2, X_3 &\sim U[0, \theta] \\ f(\underline{x}|\theta) &= \left(\frac{1}{\theta}\right)^3 \quad \theta \geq 8 \\[2mm] \xi(\theta|\underline{x}) &\propto \frac{1}{\theta^7} \quad \theta \geq 8 \\[2mm] \theta|\underline{X} &\sim \text{Pareto}(\alpha = 8, \beta = 6) \end{aligned}$$

Note, we could have actually calculated the entire pdf here:

$$\int_8^\infty \frac{1}{\theta^7} d\theta = \frac{1}{6(8)^6}$$

$$\int_8^\infty 6(8)^6 \frac{1}{\theta^7} d\theta = 1$$

$$\xi(\theta|\underline{x}) = \begin{cases} \frac{6(8)^6}{\theta^7} & \theta \geq 8 \\ 0 & \text{else} \end{cases}$$