Math 152 – Fall 2018
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warm-up # 2

Name: ________________________________

Suppose that we want to estimate the true proportion of free-throws Steph Curry can make. Unlike in class, however, we think we know a lot about his ability. From watching him play, it seems as though he only makes about three quarters of his free-throws. Your best guess, therefore, is that the true proportion of free-throws Curry can make is 0.75, with a standard deviation of 0.2. Additionally, you want to model your prior beliefs using a Beta distribution.

We get to take a random sample of size 10 free-throws (or really, he takes the free-throws). Assume the 10 free-throws have a binomial distribution where the true probability of making the basket is the unknown quantity in which we're interested. Find the posterior distribution of that quantity. (Note that it is equally valid to think about the 10 throws as Binomial or 10 separate Bernoulli measurements.)

Solution:

$$\begin{aligned}
X_1, X_2, \dots X_n &\sim \text{Bernoulli}(\theta) \\
f(\underline{x}|\theta) &= \theta^y (1-\theta)^{n-y} \qquad y = \sum_i x_i \\
\xi(\theta) &\propto \theta^{\alpha-1}(1-\theta)^{\beta-1} \qquad 0 \le \theta \le 1 \\
0.75 &= \frac{\alpha}{\alpha+\beta} \\
0.04 &= \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \\
\alpha &= 177/64 = 2.7656 \quad \beta = 59/64 = 0.9219 \\
\xi(\theta|\underline{x}) &\propto \theta^y(1-\theta)^{n-y}\theta^{2.7656-1}(1-\theta)^{.9219-1} \qquad 0 \le \theta \le 1 \\
&\propto \theta^{y+2.7656-1}(1-\theta)^{n-y+.9219-1} \qquad 0 \le \theta \le 1 \\
\theta|\underline{x} &\sim \text{Beta}(y+2.7656, 10.9219-y)
\end{aligned}$$