Math 152, Fall 2020
Jo Hardin
WU # 3
in-class: Tuesday, 9/1/20
due: Wednesday 9/2/20

Your Name: ______________________________

Names of people you worked with: ______________________________

Instructions: Work on this problem in class with your group (if you are attending class synchronously) or out of class (hopefully with a person or two! if you are attending class asynchronously). The problem should be done on a piece of paper with a pencil or on some kind of tablet. The problem should **not** by typed up or done in LaTeX.

Work for a *maximum* of 15 minutes on the problem (regardless of what time you are working). *Do not* come back to the problem to "fix it up" or "finish it." Be sure to write down the names of the people you worked with during class (or outside of class).

Take a picture of your work and use a scanning app to create a pdf (or create a pdf directly from your tablet). Upload your work to Gradescope (via Sakai) within 24 hours of class.

Task: Let θ denote the average number of defects per 100 feet of tape. Assume $X_1, X_2, \ldots X_{12} \sim$ Poisson(θ).

θ is unknown, but the prior on θ is a gamma distribution with $E[\theta] = \alpha/\beta = 2/10, \alpha = 2, \beta = 10$. When a 1200 foot roll of tape is inspected, exactly 4 defects are found.

In the interest of time, I've written out both the prior and the likelihood. You should be able to come up with these (quickly) on your own.

Find the posterior distribution of $\theta|\underline{x}$. Completely specify the distribution.

$$\begin{aligned}
\text{Prior: } \xi(\theta) &= \frac{\beta^\alpha}{\Gamma(\alpha)}\theta^{\alpha-1}e^{-\beta\theta} = \frac{10^2}{\Gamma(2)}\theta e^{-10\theta} \\
\text{Likelihood: } f(\underline{x}|\theta) &= \prod_{i=1}^{n}\frac{e^{-\theta}\theta^{x_i}}{x_i!} = \frac{e^{-n\theta}\theta^{\sum x_i}}{\prod(x_i!)}
\end{aligned}$$