

Name: _____

Let X_1, X_2, \dots, X_n be distributed $\exp(\theta)$. Find the value of θ that maximizes the joint distribution.

$$f(x|\theta) = \theta e^{-x\theta} \quad x > 0$$

Solution:

$$\begin{aligned} f(x|\theta) &= \theta e^{-x\theta} \quad x > 0 \\ f(\underline{x}|\theta) &= \theta^n e^{-\sum x_i \theta} \quad \forall x_i > 0 \\ \frac{\partial f(\underline{x}|\theta)}{\partial \theta} &= n\theta^{n-1} e^{-\sum x_i \theta} + \theta^n e^{-\sum x_i \theta} (-\sum x_i) \\ 0 &= n\theta^{n-1} e^{-\sum x_i \theta} + \theta^n e^{-\sum x_i \theta} (-\sum x_i) \\ 0 &= n + \theta(-\sum x_i) \\ \hat{\theta} &= \frac{1}{\bar{x}} \end{aligned}$$

or

$$\begin{aligned} L(\theta) &= n \ln(\theta) - \sum x_i \theta \\ \frac{\partial L(\theta)}{\partial \theta} &= \frac{n}{\theta} - \sum x_i \\ 0 &= \frac{n}{\theta} - \sum x_i \\ \hat{\theta} &= \frac{1}{\bar{x}} \end{aligned}$$

$$\frac{\partial^2 L(\theta)}{\partial \theta^2} = -\frac{n}{\theta^2} < 0$$

maximum