Math 152 – Fall 2018
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warm-up # 4

Name: ________________________________

Let $X_1, X_2, \ldots, X_n$ be distributed $\exp(\theta)$. Find the value of θ that maximizes the joint distribution of the Xs conditional on θ.

$$f(x|\theta) = \theta e^{-x\theta} \qquad x > 0$$

Solution:

$$\begin{aligned}
f(x|\theta) &= \theta e^{-x\theta} \qquad x > 0\\
f(\underline{x}|\theta) &= \theta^n e^{-\sum x_i \theta} \qquad \forall x_i > 0\\
\frac{\partial f(\underline{x}|\theta)}{\partial \theta} &= n\theta^{n-1} e^{-\sum x_i\theta} + \theta^n e^{-\sum x_i \theta}(-\sum x_i)\\
0 &= n\theta^{n-1} e^{-\sum x_i\theta} + \theta^n e^{-\sum x_i \theta}(-\sum x_i)\\
0 &= n + \theta(-\sum x_i)\\
\hat{\theta} &= \frac{1}{\overline{x}}
\end{aligned}$$

or

$$\begin{aligned}
L(\theta) &= n\ln(\theta) - \sum x_i \theta\\
\frac{\partial L(\theta)}{\partial \theta} &= \frac{n}{\theta} - \sum x_i\\
0 &= \frac{n}{\theta} - \sum x_i\\
\hat{\theta} &= \frac{1}{\overline{x}}\\
\frac{\partial^2 L(\theta)}{\partial \theta^2} &= -\frac{n}{\theta^2} < 0\\
& \text{maximum}
\end{aligned}$$