Let $X_1, X_2, \ldots, X_n$ be distributed $U[0, \theta]$. Find the maximum likelihood estimate of $\theta$.

Solution:

\[
f(x|\theta) = \begin{cases} 
\frac{1}{\theta} & 0 < x < \theta \\
0 & \text{else}
\end{cases}
\]

\[
f(x|\theta) = \frac{1^n}{\theta^n} \quad \forall x_i < \theta
\]

\[
\frac{\partial f(x|\theta)}{\partial \theta} = -n\theta^{-n-1}
\]

You can’t forget about the support of $x$ (which depends on $\theta$ !!). Remember, the function becomes zero when the very first $x_i$ gets bigger than $\theta$. Because the function is a decreasing function of $\theta$, you want $\theta$ to be bigger than all the $x$s, but no bigger than that (because then the function value would be smaller than need be). MLE of $\theta$ is $\hat{\theta} = \max(x_i)$. 