

Name: _____

Let X_1, X_2, \dots, X_n be distributed $U[0, \theta]$. Find the maximum likelihood estimate of θ .

Solution:

$$\begin{aligned} f(x|\theta) &= \frac{1}{\theta - 0} & 0 < x < \theta \\ f(\underline{x}|\theta) &= \left(\frac{1}{\theta}\right)^n & \forall x_i < \theta \\ \frac{\partial f(\underline{x}|\theta)}{\partial \theta} &= -n\theta^{-n-1} \\ 0 &= \theta \text{ ?!?!?!?!} \end{aligned}$$

You can't forget about the support of x (which depends on θ !!). Remember, the function becomes zero when the very first x_i gets bigger than θ . Because the function is a decreasing function of θ , you want θ to be bigger than all the x s, but no bigger than that (because then the function value would be smaller than need be). MLE of θ is $\hat{\theta} = \max(x_i)$.