

Name: _____

Let $X_1, X_2, \dots, X_k \sim \chi_{n_i}^2$, $i = 1, \dots, k$, independently. Show that $X_1 + X_2 + \dots + X_k = \sum_{i=1}^k X_i \sim \chi_{n_1+n_2+\dots+n_k}^2$. (Hint: use moment generating functions.)

Solution:

$$\begin{aligned}\psi_{X_i}(t) &= \left(\frac{1}{1-2t} \right)^{n_i/2} \\ Y &= \sum_{i=1}^k X_i \\ \psi_Y(t) &= E[e^{Yt}] \\ &= E[e^{t \sum_{i=1}^k X_i}] \\ &= \prod_{i=1}^k E[e^{tX_i}] \\ &= \prod_{i=1}^k \psi_{X_i}(t) \\ &= \left(\frac{1}{1-2t} \right)^{\sum n_i/2}\end{aligned}$$

See theorem 4.4.3, pg 207.