Math 152, Fall 2020
Jo Hardin
WU # 5
in-class: Tuesday, 9/8/20
due: Wednesday, 9/9/20

Your Name: _____

Names of people you worked with: _____

Instructions: Work on this problem in class with your group (if you are attending class synchronously) or out of class (hopefully with a person or two! if you are attending class asynchronously). The problem should be done on a piece of paper with a pencil or on some kind of tablet. The problem should **not** by typed up or done in LaTeX.

Work for a *maximum* of 15 minutes on the problem (regardless of what time you are working). *Do not* come back to the problem to "fix it up" or "finish it." Be sure to write down the names of the people you worked with during class (or outside of class).

Take a picture of your work and use a scanning app to create a pdf (or create a pdf directly from your tablet). Upload your work to Gradescope (via Sakai) within 24 hours of class.

Task:

The Experiment

- John Spurrier will have $n = 10$ at-bats. The random variable, X, will be his number of hits.
- Determining the prior probability: In your group, find α and β that are consistent with your prior information. Try this website: `https://isle.stat.cmu.edu/widgets/beta-distribution/`
- Comparison of the estimators:
  - $\hat{\theta}_f = \frac{X}{n} \qquad \hat{\theta}_b = \frac{X+\alpha}{n+\alpha+\beta}$
  - We use Mean Squared Error (MSE) in the frequentist sense (that is, X is the random variable, θ is no longer random) to compare estimators (apples to apples):

$$MSE(\hat{\theta}) = E[(\hat{\theta} - \theta)^2] = Var(\hat{\theta}) + bias^2(\hat{\theta}) = Var(\hat{\theta}) + [E(\hat{\theta}) - \theta]^2$$

  - Under the assumption that X has a binomial distribution with parameters 10 and θ, calculate the mean and variance of X.
  - Using the mean and variance of X, what are the variance and bias of the two estimators? What is the MSE (frequentist) for each of the two estimators? You should have two answers, both in terms of θ. Hint: the MSE for $\hat{\theta}_b$ is more messy, algebraically, than the MSE for $\hat{\theta}_f$. Let that drive your decision on which one to start with (you probably won't have enough time to find the MSE for both.)