Name: ________________________________

According to some investors, foreign stocks have the potential for high yield, but the variability in their dividends may be great. Let's say we take a random sample of 10 foreign stocks and assume that they come from a normal distribution. The data produce a sample mean of $1 dividend per share with a sample standard deviation of $2.

Find a CI for the true SD associated with dividends on foreign stocks.

Solution:

If the dividends are actually normally distributed (not really a reasonable assumption here), then theory tells us that:

$$\frac{\sum(X_i - \overline{X})^2}{\sigma^2} \sim \chi^2_9$$

Using σ to pivot (that is, putting σ in the middle of the probability statement) we get:

$$\begin{aligned}
P\left(\sum(X_i - \overline{X})^2/\chi^2_{9,.975} \leq \sigma^2 \leq \sum(X_i - \overline{X})^2/\chi^2_{9,.025} \right) &= 0.95 \\
P\left(\sqrt{\sum(X_i - \overline{X})^2/\chi^2_{9,.975}} \leq \sigma \leq \sqrt{\sum(X_i - \overline{X})^2/\chi^2_{9,.025}} \right) &= 0.95 \\
P\left(\sqrt{s^2 \cdot (n-1)/\chi^2_{9,.975}} \leq \sigma \leq \sqrt{s^2 \cdot (n-1)/\chi^2_{9,.025}} \right) &= 0.95
\end{aligned}$$

[Important!!!] where X_i (or s^2) is the random variable. (Otherwise, it doesn't make any sense to be talking about a probability.)

Therefore, the 95% confidence interval for σ is: $\left(\sqrt{s^2 \cdot (n-1)/\chi^2_{9,.975}}, \sqrt{s^2 \cdot (n-1)/\chi^2_{9,.025}}\right)$.

Note that $\chi^2_{9,.025} = 2.7$ and $\chi^2_{9,.975} = 19.02$. Giving a 95% CI for σ of:

$$\left(\sqrt{2^2 \cdot 9/19.02}, \sqrt{2^2 \cdot 9/2.7}\right)$$
$$(1.38, 3.65)$$

The correct interpretation is: we are 95% confident that the true standard deviation of dividends on foreign stocks is between $1.38 and $3.65. [Note that the method applied above is very sensitive to the normality assumption.]