Name: __________________________________________

Suppose that $X_1, X_2, \ldots, X_n$ form a random sample from a normal distribution for which both the mean $\mu$ and the variance $\sigma^2$ are unknown. Describe a method for constructing a confidence interval for $\sigma^2$ with a specified confidence coefficient.

Solution:

$$P\left(c_1 \leq \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{\sigma^2} \leq c_2\right) = 1 - \alpha$$

$$P\left(\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{c_2} \leq \sigma^2 \leq \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{c_1}\right) = 1 - \alpha$$

$$. \quad \left(\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{c_2}, \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{c_1}\right) \text{ is a } (1 - \alpha) \ 100\% \ CI \ for \ \sigma^2. \ Where \ c_1 \ is \ the \ \alpha/2 \ percentile \ of \ a \ \chi^2_{n-1} \ distribution, \ and \ c_2 \ is \ the \ 1 - \alpha/2 \ percentile \ of \ a \ \chi^2_{n-1} \ distribution.$$