Math 152 – Fall 2008
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warm-up # 7

Name: _____

Suppose that $X_1, X_2, \ldots, X_n$ form a random sample from a normal distribution for which both the mean μ and the variance σ^2 are unknown. Describe a method for constructing a confidence interval for σ^2 with a specified confidence coefficient.

Solution:

$$\begin{aligned} P\left(c_1 \leq \frac{\sum_{i=1}^n (X_i - \overline{X})^2}{\sigma^2} \leq c_2\right) &= 1-\alpha \\ P\left(\frac{\sum_{i=1}^n (X_i - \overline{X})^2}{c_2} \leq \sigma^2 \leq \frac{\sum_{i=1}^n (X_i - \overline{X})^2}{c_1}\right) &= 1-\alpha \end{aligned}$$

$\therefore$ $\left(\frac{\sum_{i=1}^n (X_i-\overline{X})^2}{c_2}, \frac{\sum_{i=1}^n (X_i-\overline{X})^2}{c_1}\right)$ is a $(1-\alpha)$ 100% CI for σ^2. Where c_1 is the $\alpha/2$ percentile of a χ^2_{n-1} distribution, and c_2 is the $1-\alpha/2$ percentile of a χ^2_{n-1} distribution.