

Name: _____

Suppose $X \sim \text{Poisson}(\theta)$. $f(x|\theta) = \frac{e^{-\theta}\theta^x}{x!}$. Find the Fisher Information in X .

Solution:

$$\begin{aligned}\lambda(x|\theta) &= \ln f(x|\theta) = -\theta + x \ln \theta - \ln(x!) \\ \lambda'(x|\theta) &= -1 + x/\theta \\ \lambda''(x|\theta) &= -x/\theta^2\end{aligned}$$

$$\begin{aligned}I(\theta) &= -E[\lambda''(X|\theta)] = -E[-X/\theta^2] \\ &= \theta/\theta^2 = 1/\theta\end{aligned}$$

$$\begin{aligned}I(\theta) &= \text{var}(\lambda'(X|\theta)) = \text{var}(-1 + X/\theta) \\ &= \frac{1}{\theta^2} \text{var}(X) = \theta/\theta^2 = 1/\theta\end{aligned}$$

$$\begin{aligned}I(\theta) &= E\{[\lambda'(X|\theta)]^2\} \\ &= E[1 - 2X/\theta + X^2/\theta^2] \\ &= 1 - \frac{2}{\theta}E[X] + \frac{1}{\theta^2}E(X^2) \\ &= 1 - \frac{2}{\theta}\theta + \frac{1}{\theta^2}(\text{var}(X) + E[X]^2) \\ &= 1 - 2 + \frac{1}{\theta^2}(\theta + \theta^2) = 1/\theta\end{aligned}$$

Notice that the information in θ depends on θ . This isn't always true.