Math 152 – Fall 2018
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warm-up # 8

Name: ________________________________

Let's say we are trying to estimate the total number of soft drinks a particular vending machine will sell in a typical week. We want to find a 90% posterior interval for μ. Our prior information (e.g., from past weeks) tells us:

$$\begin{aligned} \mu|\tau &\sim N(750, 5/\tau = \frac{1}{(1/5)\tau}) \\ \tau &\sim gamma(1, 45) \end{aligned}$$

$\mu_0 = 750$, $\lambda_0 = 1/5$, $\alpha_0 = 1$, $\beta_0 = 45$
Our random sample of 10 weeks gives $\overline{x} = 692$ and $s^2 = \frac{14400}{9} = 1600$.

Find a 90% posterior interval for μ. (And interpret.)

Solution:

Our posterior parameters are:

$$\begin{aligned} \mu_1 &= \frac{\lambda_0\mu_0 + n\overline{x}}{\lambda_0 + n} = \frac{(1/5)750 + 6920}{(1/5) + 10} = 693.14 \\ \lambda_1 &= \lambda_0 + n = 10.2 \\ \alpha_1 &= \alpha_0 + n/2 = 1 + 5 = 6 \\ \beta_1 &= \beta_0 + \frac{1}{2}\sum_{i=1}^{n}(x_i - \overline{x})^2 + \frac{n\lambda_0(\overline{x} - \mu_0)^2}{2(\lambda_0 + n)} = 45 + 14400/2 + \frac{10(1/5)(692 - 750)^2}{2(1/5 + 10)} = 7574.8 \end{aligned}$$

To find a 90% PI, we need to find cutoff values such that $P(-c \leq t_{2\alpha_1} \leq c) = 0.9$. $2\alpha_1 = 12$, $P(t_{12} \leq 1.782) = 0.95$.

$$\begin{aligned} P(-1.782 \leq U \leq 1.782) &= 0.9 \\ P(\mu_1 - 1.782(\frac{\beta_1}{\lambda_1\alpha_1})^{1/2} \leq \mu \leq \mu_1 + 1.782(\frac{\beta_1}{\lambda_1\alpha_1})^{1/2}) &= 0.9 \\ P(673.31 \leq \mu \leq 712.97) &= 0.9 \end{aligned}$$

Given our prior beliefs and data, there is a 90% probability that the average number of cans sold per week is between 673.31 and 712.97 cans.