Math 152 – Fall 2008
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warm-up # 9

Name: _____

Suppose your prior distribution for θ, the proportion of Californians who support the death penalty is Beta with mean 0.6 and standard deviation 0.3. A random sample of 1000 Californians is taken, and 650 support the death penalty. Give a 95% posterior interval for θ. (Say something about the death penalty in your PI interpretation.)

Solution: Your prior information gives you:

$$\begin{aligned} 0.6 &= \frac{\alpha}{\alpha+\beta} \\ 0.09 &= \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \\ \alpha &= 1, \beta = 2/3 \end{aligned}$$

$$\begin{aligned} Y &\sim \text{Binomial}(\theta, 1000) \\ f(y|\theta) &= \binom{1000}{y}\theta^y(1-\theta)^{n-y} \\ \xi(\theta) &\propto \theta^{\alpha-1}(1-\theta)^{\beta-1} \qquad 0 \le \theta \le 1 \\ \xi(\theta|y) &\propto \theta^y(1-\theta)^{n-y}\theta^{1-1}(1-\theta)^{2/3-1} \qquad 0 \le \theta \le 1 \\ &\propto \theta^{y+1-1}(1-\theta)^{n-y+2/3-1} \qquad 0 \le \theta \le 1 \\ \theta|y &\sim \text{Beta}(651, 350.67) \end{aligned}$$

(Notice how the prior information has very little effect when the sample size is so large. Well, and also it's because your prior standard deviation is so large.)

First we need to find c_1 and c_2 from the Beta(651,350.67) distribution. Let $X \sim$ Beta(651, 350.67). Find c_1 and c_2 such that:

$$\begin{aligned} P(X < c_1) = 0.025 \qquad c_1 = 0.620 \\ P(X \le c_2) = 0.975 \qquad c_2 = 0.679 \end{aligned}$$

We know then that the 95% posterior interval for θ is $(c_1, c_2) = (0.62, 0.68)$. There is a 0.95 probability that the true proportion of Californians who support the death penalty is between c_1 and c_2 (between 62% and 68%).