

Name: _____

Let's say we're going to flip a coin 100 times, and we assume the probability of heads is $\theta = 0.5$.

1. Use the normal approximation to the binomial to calculate the probability of getting 55 or more heads.
2. For what value of c is the probability of c or more heads no more than 0.05?

Solution

1.

$$\begin{aligned}\pi(\theta = 0.5|\delta) &= P(X > 55|\theta = 0.5) \\ &= P\left(Z > \frac{55 - n\theta}{\sqrt{n\theta(1 - \theta)}}\right) \\ &= P\left(Z > \frac{55 - 50}{\sqrt{25}}\right) \\ &= P(Z > 1) = 1 - P(Z \leq 1) = 0.1587\end{aligned}$$

2.

$$\begin{aligned}\pi(\theta_0 = 0.5|\delta) &= P(X > c|\theta = 0.5) \\ &\leq 0.05 \\ P(X < c|\theta = 0.5) &\geq 0.95 \\ P\left(Z < \frac{c - 50}{5}|\theta = 0.5\right) &\geq 0.95 \\ \frac{c - 50}{5} &\geq 1.645 \\ c &\geq 58.25\end{aligned}$$

We let c be as small as possible, so $c=58.25$ (so you'd have to get at least 59 heads).