Math 154, Fall 2017, WU #6 (SVM)

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November 13, 2017

Name: ____

Consider the following 5 data points in R^2.

1. Draw the points in R^2 and argue that the values are not linearly separable.
2. Transform the points using the function $\phi : R^2 \rightarrow R^2$. Plot the data again, this time using the first and second coordinate of the transformed data.

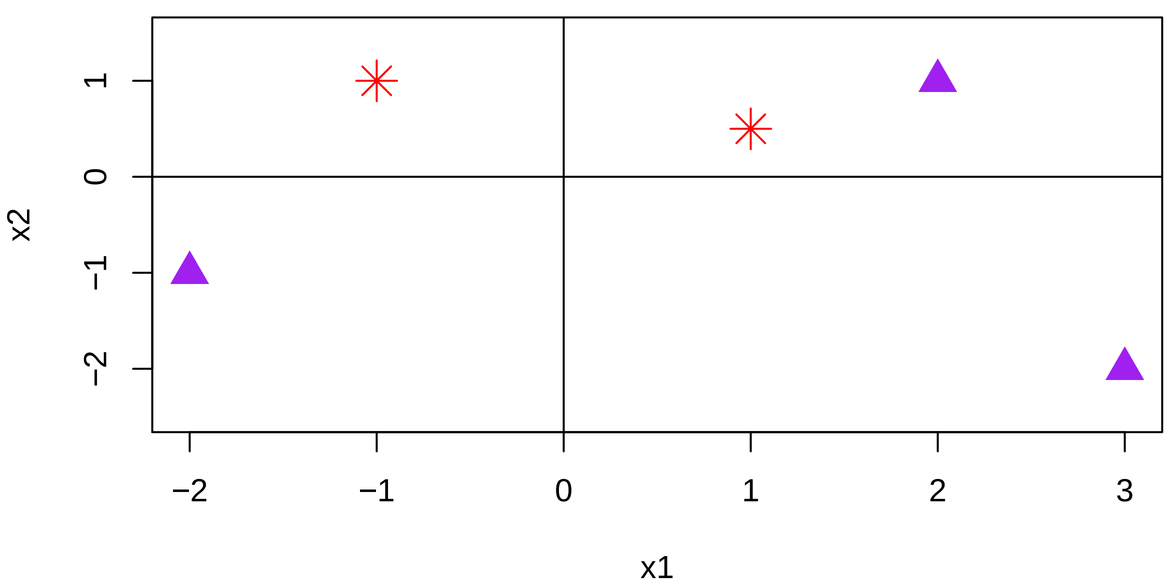
$$\phi(x_1, x_2) = (x_1^2, x_2)$$

3. Sketch the optimal separating hyperplane and the maximal margin on the second figure.

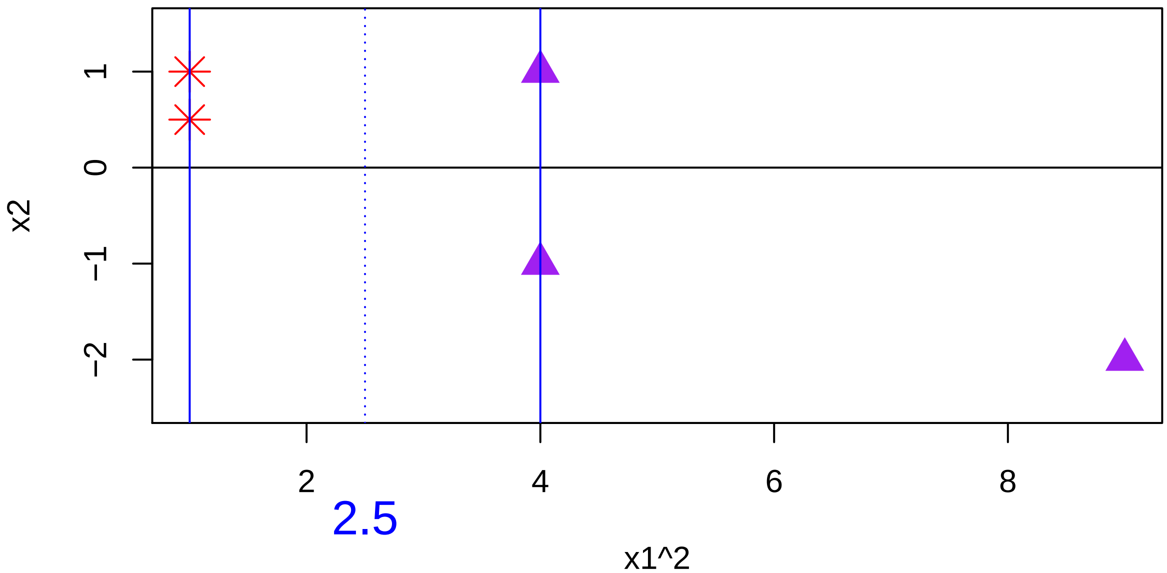
point	x1	x2	class
1	-1	1	red
2	1	0.5	red
3	2	1	purple
4	-2	-1	purple
5	3	-2	purple

Solution:

In the original space:



In the transformed space with the separating hyperplane:



Note that the $\mathbf{w}$ vector here is the one such that $\mathbf{w} \cdot \mathbf{u} + b \geq 0$ assigns to red. Therefore,

$$
\begin{aligned}
x1^2 &\geq 2.5 \\
\iff x1^2 - 2.5 &\geq 0 \\
\iff x1^2 \times 1 + x2 \times 0 - 2.5 &\geq 0 \\
\iff (x1^2, x2) \cdot (1, 0) - 2.5 &\geq 0
\end{aligned}
$$

So $\mathbf{w} = (1, 0)$ and $b = -2.5$.

Back in the original space with the decision boundary