Math 158 – Spring 2009
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warm-up # 6

Name: ________________________________

Consider the linear model:

$$
\begin{aligned}
Y_{ij} &= \mu_i + \epsilon_{ij} \\
Y_{ij} &= \text{response for } j^{th} \text{ observation in } i^{th} \text{ treatment group} \\
\mu_i &= \text{population average response for } i^{th} \text{ group} \\
\epsilon_{ij} &= \text{model error term / true error for } j^{th} \text{ observation in } i^{th} \text{ treatment group} \\
i &= 1, 2, \ldots, r \\
j &= 1, 2, \ldots, n_i
\end{aligned}
$$

Use the least squares criterion to estimate μ_i.
Solution:

$$
\begin{aligned}
Q &= \sum_{i=1}^{r}\sum_{j=1}^{n_i}(Y_{ij} - \mu_i)^2 \\
Q_i &= \sum_{j=1}^{n_i}(Y_{ij} - \mu_i)^2 \\
\frac{\partial Q}{\partial \mu_i} &= \frac{\partial Q_i}{\partial \mu_i} \\
&= \frac{\partial \sum_{j=1}^{n_i}(Y_{ij} - \mu_i)^2}{\partial \mu_i} \\
&= -2\sum_{j=1}^{n_i}(Y_{ij} - \mu_i) \\
\frac{\partial Q}{\partial \mu_i} &= 0 \\
\rightarrow \hat{\mu}_i &= \frac{1}{n_i}\sum_{j=1}^{n_i} Y_{ij} \\
&= \overline{Y}_{i\cdot}
\end{aligned}
$$