

Differentiation Formulas

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| 1. $(f(x) \pm g(x))' = f'(x) \pm g'(x)$ | 2. $(kf(x))' = kf'(x)$ |
| 3. $(f(x)g(x))' = f(x)g'(x) + g(x)f'(x)$ | 4. $\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$ |
| 5. $(f(g(x)))' = f'(g(x)) \cdot g'(x)$ | 6. $\frac{d}{dx}(x^n) = nx^{n-1}$ |
| 7. $\frac{d}{dx}(e^x) = e^x$ | 8. $\frac{d}{dx}(a^x) = a^x \ln a \quad (a > 0)$ |
| 9. $\frac{d}{dx}(\ln x) = \frac{1}{x}$ | |
| 10. $\frac{d}{dx}(\sin x) = \cos x$ | 11. $\frac{d}{dx}(\cos x) = -\sin x$ |
| 12. $\frac{d}{dx}(\tan x) = \frac{1}{\cos^2 x}$ | |
| 13. $\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$ | 14. $\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$ |

A Short Table of Indefinite Integrals

I. Basic Functions

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| 1. $\int x^n dx = \frac{1}{n+1}x^{n+1} + C, \quad n \neq -1$ | 5. $\int \sin x dx = -\cos x + C$ |
| 2. $\int \frac{1}{x} dx = \ln x + C$ | 6. $\int \cos x dx = \sin x + C$ |
| 3. $\int a^x dx = \frac{1}{\ln a}a^x + C, \quad a > 0$ | 7. $\int \tan x dx = -\ln \cos x + C$ |
| 4. $\int \ln x dx = x \ln x - x + C$ | |

II. Products of e^x , $\cos x$, and $\sin x$

8. $\int e^{ax} \sin(bx) dx = \frac{1}{a^2 + b^2} e^{ax} [a \sin(bx) - b \cos(bx)] + C$
9. $\int e^{ax} \cos(bx) dx = \frac{1}{a^2 + b^2} e^{ax} [a \cos(bx) + b \sin(bx)] + C$
10. $\int \sin(ax) \sin(bx) dx = \frac{1}{b^2 - a^2} [a \cos(ax) \sin(bx) - b \sin(ax) \cos(bx)] + C, \quad a \neq b$
11. $\int \cos(ax) \cos(bx) dx = \frac{1}{b^2 - a^2} [b \cos(ax) \sin(bx) - a \sin(ax) \cos(bx)] + C, \quad a \neq b$
12. $\int \sin(ax) \cos(bx) dx = \frac{1}{b^2 - a^2} [b \sin(ax) \sin(bx) + a \cos(ax) \cos(bx)] + C, \quad a \neq b$

III. Product of Polynomial $p(x)$ with $\ln x$, e^x , $\cos x$, $\sin x$

13. $\int x^n \ln x dx = \frac{1}{n+1}x^{n+1} \ln x - \frac{1}{(n+1)^2}x^{n+1} + C, \quad n \neq -1$
14.
$$\int p(x)e^{ax} dx = \frac{1}{a}p(x)e^{ax} - \frac{1}{a} \int p'(x)e^{ax} dx$$

$$= \frac{1}{a}p(x)e^{ax} - \frac{1}{a^2}p'(x)e^{ax} + \frac{1}{a^3}p''(x)e^{ax} - \dots$$

(+ - + - ...)

(signs alternate)

$$15. \int p(x) \sin ax \, dx = -\frac{1}{a} p(x) \cos ax + \frac{1}{a} \int p'(x) \cos ax \, dx$$

$$= -\frac{1}{a} p(x) \cos ax + \frac{1}{a^2} p'(x) \sin ax + \frac{1}{a^3} p''(x) \cos ax - \dots$$

(- + + - - + + ...)

(signs alternate in pairs after first term)

$$16. \int p(x) \cos ax \, dx = \frac{1}{a} p(x) \sin ax - \frac{1}{a} \int p'(x) \sin ax \, dx$$

$$= \frac{1}{a} p(x) \sin ax + \frac{1}{a^2} p'(x) \cos ax - \frac{1}{a^3} p''(x) \sin ax - \dots$$

(+ + - - + + - - ...)

IV. Integer Powers of $\sin x$ and $\cos x$

$$17. \int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx, \quad n \text{ positive}$$

$$18. \int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx, \quad n \text{ positive}$$

$$19. \int \frac{1}{\sin^m x} \, dx = \frac{-1}{m-1} \frac{\cos x}{\sin^{m-1} x} + \frac{m-2}{m-1} \int \frac{1}{\sin^{m-2} x} \, dx, \quad m \neq 1, m \text{ positive}$$

$$20. \int \frac{1}{\sin x} \, dx = \frac{1}{2} \ln \left| \frac{(\cos x) - 1}{(\cos x) + 1} \right| + C$$

$$21. \int \frac{1}{\cos^m x} \, dx = \frac{1}{m-1} \frac{\sin x}{\cos^{m-1} x} + \frac{m-2}{m-1} \int \frac{1}{\cos^{m-2} x} \, dx, \quad m \neq 1, m \text{ positive}$$

$$22. \int \frac{1}{\cos x} \, dx = \frac{1}{2} \ln \left| \frac{(\sin x) + 1}{(\sin x) - 1} \right| + C$$

23. $\int \sin^m x \cos^n x \, dx$: If m is odd, let $w = \cos x$. If n is odd, let $w = \sin x$. If both m and n are even and non-negative, convert all to $\sin x$ or all to $\cos x$ (using $\sin^2 x + \cos^2 x = 1$), and use IV-17 or IV-18. If m and n are even and one of them is negative, convert to whichever function is in the denominator and use IV-19 or IV-21. If both m and n are even and negative, substitute $w = \tan x$.

V. Quadratic in the Denominator

$$24. \int \frac{1}{x^2 + a^2} \, dx = \frac{1}{a} \arctan \frac{x}{a} + C, \quad a \neq 0$$

$$25. \int \frac{bx + c}{x^2 + a^2} \, dx = \frac{b}{2} \ln|x^2 + a^2| + \frac{c}{a} \arctan \frac{x}{a} + C, \quad a \neq 0$$

$$26. \int \frac{1}{(x-a)(x-b)} \, dx = \frac{1}{a-b} (\ln|x-a| - \ln|x-b|) + C, \quad a \neq b$$

$$27. \int \frac{cx + d}{(x-a)(x-b)} \, dx = \frac{1}{a-b} [(ac + d) \ln|x-a| - (bc + d) \ln|x-b|] + C, \quad a \neq b$$

VI. Integrands Involving $\sqrt{a^2 + x^2}$, $\sqrt{a^2 - x^2}$, $\sqrt{x^2 - a^2}$, $a > 0$

$$28. \int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \arcsin \frac{x}{a} + C$$

$$29. \int \frac{1}{\sqrt{x^2 \pm a^2}} \, dx = \ln \left| x + \sqrt{x^2 \pm a^2} \right| + C$$

$$30. \int \sqrt{a^2 \pm x^2} \, dx = \frac{1}{2} \left(x \sqrt{a^2 \pm x^2} + a^2 \int \frac{1}{\sqrt{a^2 \pm x^2}} \, dx \right) + C$$

$$31. \int \sqrt{x^2 - a^2} \, dx = \frac{1}{2} \left(x \sqrt{x^2 - a^2} - a^2 \int \frac{1}{\sqrt{x^2 - a^2}} \, dx \right) + C$$