

Name:

Calculate the following area:

$$\int_{-1}^1 \frac{1}{x^4} dx$$

Solution:

The trouble spot is at $x = 0$, so we write:

$$\int_{-1}^1 \frac{1}{x^4} dx = \int_{-1}^0 \frac{1}{x^4} dx + \int_0^1 \frac{1}{x^4} dx$$

However, both integrals diverge. For example,

$$\begin{aligned} \int_0^1 \frac{1}{x^4} dx &= \lim_{a \rightarrow 0^+} \int_a^1 \frac{1}{x^4} dx \\ &= \lim_{a \rightarrow 0^+} \left. -\frac{x^{-3}}{3} \right|_a^1 \\ &= \lim_{a \rightarrow 0^+} \left(\frac{1}{3a^3} - \frac{1}{3} \right) \end{aligned}$$