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WU # 3

Name: _____

In a recent archaeological expedition, a scroll was discovered containing a description of a plan to build what appears to be the Tower of Babel. According to the manuscript, the tower was supposed to have a circular cross section and "go up to the heavens" (i.e., be infinitely high). A mathematician was consulted to solve some of the questions posed by the archaeologists. The mathematicians plotted half of the silhouette of the tower on a set of coordinate axis with the y-axis running through the entire center, and discovered that it was approximated by the curve $y = -100\ln(x/5)$. Would such a tower have finite volume? [If so, find the volume.]

Solution:
If we slice horizontally, the volume of one disk with radius x and thickness dy is $\pi(x^2)dy$. We know $y = -100\ln(x/5)$, so $\frac{x}{5} = e^{-y/100}$. Consequently,

$$\begin{aligned}
\text{volume} &= \int_0^\infty \pi(5e^{-y/100})^2 dy \\
&= \pi\int_0^\infty 25e^{-y/50}dy \\
&= \lim_{b\to\infty} \pi\int_0^b 25e^{-y/50}dy \\
&= \lim_{b\to\infty} -1250\pi e^{-y/50}\Big|_0^b \\
&= -1250\pi(\lim_{b\to\infty} e^{-b/50} - e^0) \\
&= 1250\pi
\end{aligned}$$

Note: the manuscript mentioned that 4200 cubic "shrims" (Babel's unit of length) of stone were available to build the tower. The base of the tower was to have radius 4 shrims. Did they have enough?

If we had done the above integral with shrims, then we would have found the volume of a tower with a radius of 5 shrims at the base [that is because $-100\ln(x/5)$ passes through the x-axis at $x = 5$]. Its volume would be 1250π shrims (which is less than 4200 shrims). The tower with a base of radius 4 shrims has less volume than ours, so yes, they would have had enough stone.