Math 58B – Intro to Biostatistics
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CLT examples

Example 1 Birth weights of babies in the US have been said to be reasonably modeled by a normal distribution with mean 3250 grams and a standard deviation of 550 grams, $Y \sim N(3250, 550)$. Babies weighing less than 2500 grams are considered to be of low birth weight.

1. Draw a sketch of the distribution of the birth weights
2. On the graph, shade the area of the distribution corresponding to babies of low birth weight. What is the probability that a randomly selected baby will be of low birth weight?
3. What is the probability that a randomly selected baby weighs more than 4536 grams ($\approx$ 10 lbs)?
4. What is the probability that a randomly selected baby weighs between 2500 and 4536 grams?
5. What would a baby have to weigh to be in the heaviest 2.5% of all weights? 1%? lightest?
6. How much would a baby have to weigh to be in the middle 80% of weights?

The Central Limit Theorem

The central limit theorem says that for certain statistics (averages or sums, really), the sampling distribution of those statistics will be normal **regardless** of the underlying population distribution... as long as the sample size is big enough. Typically, we need $n \geq 30$ for sample averages and $np \geq 5/10, n(1-p) \geq 5/10$ in the binomial setting.

$$\begin{aligned} \overline{Y} &\sim N(\mu, \sigma/\sqrt{n}) \\ \hat{p} &\sim N(p, \sqrt{p(1-p)/n}) \\ X &\sim N(np, \sqrt{np(1-p)}) \end{aligned}$$

Example 2 Continuing with the birth weight example... data from the *National Vital Statistics Report* indicate that there were 3,880,894 births in the US in 1997, and a total of 291,154 babies were of low birth weight. Consider a random sample of 100 babies.

1. Describe the sampling distribution of the average weight of 100 randomly selected babies. Draw the curve.
2. Describe the sampling distribution of the proportion of low weight babies in a sample of 100 randomly selected babies. Draw the curve.
3. Describe the sampling distribution of the number of low weight babies in a sample of 100 randomly selected babies. Draw the curve.
4. What is the probability that in a sample of 100 babies, half of them would be of low birth weight?
5. Where would you expect 80% of your sample averages to fall?