

# Women in Mathematics Symposium

February 20, 2010

University of Southern California

## Titles and Abstracts

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**Asuman Aksoy, CMC**

*Some Results on Metric Trees*

The study of injective envelopes of metric spaces, also known as metric trees (R-trees or T-theory), has its motivation in many subdisciplines of mathematics as well as biology, medicine and computer science. A metric tree is a metric space  $(M,d)$  such that for every  $x,y$  in  $M$  there is a unique arc between  $x$  and  $y$  and this arc is isometric to an interval in  $\mathbb{R}$  ([4],[5]). In this talk, we examine convexity and compact structures in metric trees ([2],[3]), and show that nonempty closed convex subsets of a metric tree enjoy many properties shared by convex subsets of Hilbert spaces and admissible subsets of hyperconvex spaces [1].

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**Alethea Barbaro, UCLA**

*Agent-based models of social dynamics*

In my research, I focus on studying and simulating the dynamics of groups of individuals. I will speak about using agent-based mathematical models to simulate large groups of social individuals. I will discuss the work I have done modeling fish migration and also current work on gang dynamics and territorial behavior in Los Angeles.

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**Joanna A. Bieri, University of Redlands**

*Edge Flames in Narrow Channels*

There has been a great deal interest in the development of mini-combustor systems for use in powering or heating small devices. In the past, it was widely thought that combustion was impossible in such confined geometries, but recent experiments have shown that not only is combustion possible, but rich dynamics are seen in the case of a flame confined in a very narrow channel. The mechanisms that govern the dynamics of these flames are not well understood. By mathematically modeling the dynamics of an edge-flame confined in a narrow channel, we are able to identify the mechanisms responsible for flames' behavior and identify situations in which flame stability can be ensured. This talk will discuss a mathematical model for an edge flame in a narrow channel, including heat loss conditions at the channel wall, and numerical methods used to solve and predict flame shape and behavior.

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**Maria Isabel Bueno Cachadina, University of California, Santa Barbara**

*Maximum exponent of boolean circulant matrices with fixed number of nonzero entries in its generating vector.*

It is well known that the maximum exponent that an  $n$ -by- $n$  boolean primitive circulant matrix can attain is  $n-1$ . We find the maximum exponent that  $n$ -by- $n$  boolean primitive circulant matrices with constant number of nonzero entries in its generating vector can attain. We also give matrices attaining such exponents. Solving this problem we also solve two equivalent problems: 1) find the maximum exponent attained by primitive Cayley digraphs on a cyclic group whose vertices have constant outdegree; 2) determine the maximum order of basis for  $Z_n$  with fixed cardinality.

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**Jen-Mei Chang, Cal State Long Beach**

*Applying Image Processing Techniques to Promote Conceptual Understanding in Linear Algebra Classes*

The Intro to Linear Algebra at Long Beach State is traditionally populated with math and computer science majors. Recently, we have observed an increasing number of students from other disciplines partake in the course. These students are motivated by the intention to advance their collective knowledge in math through the learning of linear algebra. Inevitably, we are witnessing a pressing need to deliver innovative instructions that promote understandings of intellectually challenging concepts to a diverse audience. In this presentation, we will share a few image processing techniques that can be used in linear algebra instruction to illustrate concepts such as matrix multiplications and orthogonal projection. In particular, we frontload the lessons with practical use of linear algebra ideas to increase students' interest level and offer a reference system for abstract concepts. This application-first, theory-second approach was shown to be effective in consolidating mental images and generating big pictures of difficult concepts. Albeit a slight initial resistance, students were comfortable using some of the techniques learned in the class in solving real-life problems in the end-of-semester poster presentation.

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**Sarah Eichhorn, UC Irvine**

*Cycloidal cracks on Europa*

Jupiter's moon Europa is of great interest to planetary researchers due to its liquid water ocean beneath the surface ice shell. Europa has some unique surface features including cycloid shaped cracks. We will pose a mathematical model for tidal stresses and illustrate how this model can be used to predict the formation of the cycloidal cracks. A new modified Numerov-Cooley type numerical method for solving the resulting nonlinear integro-differential equation will also be presented.

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**Jessica Grossbard, California State University, Los Angeles**

*The Fractional Chromatic Number of Integer Distance Graphs*

Given a set  $M$  of positive integers, an integer distance graph has the set of all integers as its vertex set with an edge joining two vertices  $u$  and  $v$  if and only if  $|u-v| \in M$ . The problem of determining (or estimating) certain parameters of these graphs is closely related to the number theory problem of finding the maximum density,  $\mu(M)$ , of a set  $M$  of integers with missing differences. We study  $\mu(M)$  and  $\kappa(M)$ , the parameter of the Lonely Runner Conjecture, for the cases where the elements of  $M$  are in arithmetic progression, geometric progression, and as well as when  $M$  is a general three element set  $\{i, j, k\}$ . This allows us to determine the fractional chromatic number and find bounds for the circular chromatic number for the related distance graphs.

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**Mihaela Ignatova, USC**

*Spatial Complexity of Solutions to Parabolic Differential Equations with Gevrey Coefficients*

We study the nodal (zero) sets of the 1-periodic in space solutions  $u(x,t)$  of the 1D second order parabolic PDE  $u_t - u_{xx} + v(x,t)u_x + w(x,t)u=0$  with Gevrey coefficients in  $G^{\{\sigma\}}$  with  $\sigma \geq 1$ . In order to obtain an upper bound on the size of the nodal set  $\{x \in [0,1]: u(x,t)=0\}$ , the solutions of the PDE have to satisfy the strong unique continuation property (SUCP); that is, a solution which vanishes at an infinite order at a point is identically zero. We prove that the SUCP is satisfied for the range of exponents  $\sigma$  which includes non-analytic Gevrey classes. In particular, we give an upper bound on the size of the zero set of solutions with a polynomial dependence on the coefficients.

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**Nishu Lal, UC Riverside**

*Spectral Analysis on Self-similar Sets*

The Laplacian operator is one of the most important operators studied in the theory of analysis on manifolds. To define a differential operator like the Laplacian on fractals is not possible from the classical viewpoint of analysis. We construct the Laplacian on finitely ramified self-similar fractals, such as the Sierpinski gasket and discuss its spectrum. The decimation method is a process that describes the relationship between the spectrum of the Laplace operator and the dynamics of the iteration of a certain polynomial on  $\mathbb{C}$ . Furthermore, we discuss the spectral zeta function of the Laplacian. Teplyaev discovered the product structure of the spectral zeta function in the case of Sierpinski gasket that involves a geometric part and a new zeta function of a polynomial induced by the decimation method. An interesting feature of the product structure is the cancellation phenomenon between the poles of the zeta function of a polynomial and the zeros of the geometric part of the spectral zeta function of the Laplacian. Initially, M. Lapidus illustrated a similar product structure for self-similar fractal strings.

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**Rena Levitt, Pomona College**

*Combinatorial Geodesics in Triangle-Square Complexes*

In a cellular complex, the combinatorial metric on the 0-skeleton is defined by taking the distance between vertices  $v$  and  $w$  to be the minimum length of edge paths between  $v$  and  $w$ . Paths of minimal length are combinatorial geodesics. In this talk, I will discuss the structure of combinatorial geodesics in non-positively curved triangle-square complexes, and how they differ from paths in pure complexes.

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**Kate Longo, UC Irvine**

*A new fourth order PDE for image processing*

Much research has been done studying nonlinear diffusions as a method for noise removal, a fundamental problem in image processing. Particularly, the second order Perona-Malik equation was considered a breakthrough in noise removal. More recently, fourth order diffusion equations have been considered in an attempt to overcome some shortcomings of the Perona-Malik model. I will present some background on the use of diffusion equations for noise removal, give an overview of the work that has been done on fourth order equations, and discuss a new fourth order model, which is an improvement upon existing methods.

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**Susan Montgomery, USC**

Keynote address. *Non-commutativity: from physics to groups and Hopf Algebras*

When we are learning mathematics, most students come to believe that multiplication should be commutative, that is  $ab = ba$  for all elements in the set they are looking at. The first example of non-commutativity we see is usually matrix multiplication.

In this talk I hope to show that non-commutativity arises "in nature" by surveying some classical examples such as the Weyl algebra (which arose in quantum mechanics), the braid group, the fundamental group of a space, and at the end, quantum groups and the idea of braided categories.

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**Anna Varvak, Soka University of America**

*Problems with "problem solving"*

Learning problem solving is one of the central objectives for mathematics classes in elementary and secondary schools. It is probably the least controversial objective for a math class; everyone agrees that it's one of the reasons why everyone should learn math. Yet US students tend to perform very poorly on tests for problem solving in mathematical contexts, especially in contexts requiring multiple steps.

One of the reasons probably lies in the tremendous disconnect between real-life problem solving and those traditional representations of problem-solving within mathematical context that are popularly known as "word problems", together with the specialized procedural strategies that tend to be taught in math classes (such as looking for key words to translate the scenario into an algebraic equation).

Training teachers, who themselves may not have experienced any other educational scenarios requiring mathematical problem solving, to try more meaningful problem-solving sessions poses many interesting challenges. In this talk, I will discuss my ongoing collaboration with the Riverbend Community Math Center (based in South Bend, Indiana) in training elementary and middle-school teachers to conduct meaningful, regular, in-depth problem-solving sessions (half-hour for one or two multi-step problems, ideally once or twice a week). I hope to get feedback on our intended method of assessment of the results of this past summer's training and its impact on the students' performance on the mathematical problem solving portion on ISTEPS (Indiana's state-wide examination of student performance), as well as getting the teachers to share about the challenges they faced in implementing such sessions.

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**Ursula Whitcher, Harvey Mudd College**

*Periods, Picard-Fuchs Equations, and Elliptic Curves*

Algebraic geometers CAN tell the difference between their doughnuts and their coffee cups; in fact, they can even distinguish between different sizes of doughnut! We will use path integrals (called periods) and differential equations (called Picard-Fuchs equations) to describe the behavior of a family of complex tori (called elliptic curves, or possibly doughnuts). If time permits, we will sketch the way similar techniques can be used to study families of higher-dimensional varieties.

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**Eliana Zoque, UC Riverside**

*On the variety of almost commuting nilpotent matrices.*

We study the variety of  $n$  by  $n$  matrices with commutator of rank at most one. We describe its irreducible components; two of them correspond to the pairs of commuting matrices, and  $n-2$  components of smaller dimension corresponding to the pairs of rank one commutator.

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