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Course Description **Pomona Math 171** **Abstract Algebra I: Groups & Rings**

What is Abstract Algebra? Is there a formula—similar to the quadratic formula—for solving fifth degree algebraic equations? How do you go about finding all the integer solutions to an equation like $y^3 = x^2 + 4$? Abstract Algebra has its roots in attempts to answer these classical problems, but its methods and techniques permeate all of modern mathematics. Modern abstract algebra is organized around the axiomatic study of algebraic structures. Vector spaces—that you studied in linear algebra—are an example of an algebraic structure, but there are many more. All algebraic structures have common features, but each has its own distinct personality and its own areas of application. This first semester of abstract algebra will be devoted to the study of groups and rings. Groups are mathematicians way of capturing and analyzing symmetry while rings allow for a thorough study of arithmetical operations. The course develops groups and rings axiomatically and from first principles. While, we will prove everything along the way, we will also spend time developing mathematical intuition for these algebraic structures. From very meager beginnings, we will develop a powerful theory. My hope is that, by the end of the semester, you will have an appreciation for the depth, beauty, and power of this area of mathematics. Watch A1: What is Abstract Algebra? for a brief intro to the topic.

Text. We will cover a good bit of Chapters 1–12 and 15–18 of the following text:

> Shahriar Shahriari, *Algebra in Action, A course in groups, rings, and fields*, American Mathematical Society, 2017.

Sample Questions. Here are three problems that you will be to do by the end of the course.

- Describe all integer solutions to $y^3 = x^2 + 4$.
- In how many ways can you color the faces of a cube using 47 colors? The faces of the cube are identical, you can use the same color more than once, and if you can get from one coloring to another by rotating the cube, then count the two colorings as the same.
- You have a set of 169 one to one and onto functions from a set X to itself. Assume that the composition of any pair of the functions is also a function among your set. Prove that $f(g(x)) = g(f(x))$ for every $x \in X$ and every pair of functions f and g in the set.

Who should take Math 171? Abstract Algebra is one of the core areas of modern mathematics, and the language and methods of abstract algebra are present in every area of mathematics. Math 171 is a required course in the pure mathematics track for the math major at Pomona College, and while it is not required in the other tracks, the study of abstract algebra can enhance the breadth of your mathematical experience. Students outside of mathematics who are interested in theoretical computer science or theoretical physics will also benefit from a background in abstract algebra. Math 171 is also a prerequisite for more advanced algebra courses such as Galois Theory, Representation Theory, Algebraic Geometry, Algebraic Number Theory, and so on. Graduate schools in pure mathematics expect at least one—and often two—abstract algebra courses as part of a student's undergraduate course work. Graduate schools in applied mathematics, statistics, operations research, computer science, or theoretical phyiscs do not require such a course, but often do see the successful completion of an abstract algebra course as a sign of mathematical maturity.

What are the prerequisites for Math 171? The formal prerequisite for Math 171 is Linear Algebra. However, we strongly urge a transition level class (i.e., Combinatorics, Number Theory & Cryptography, Intro to Analysis) before taking Math 171. You will need to have an exposure to rigorous mathematical arguments before taking Math 171. While it is not necessary to be completely at home deciphering and writing proofs—you will get plenty of experience in Math 171—this class should not be your first exposure to rigorous mathematics. Since the material is developed from the ground up, I don't need you to remember many facts or theorems from other classes. But the course is fast-paced and you will need some prior experience in deciphering and writing mathematical arguments.

Are there overlaps with other classes? There is a little bit of overlap with elementary Number Theory. Math 113 (Number Theory & Cryptography) is possibly a good preparation for Math 171, but once you have completed Math 171, taking Math 113 is not recommended.

When should I take Abstract Algebra? At Pomona, we teach Math 171 every spring. Most students who take Abstract Algebra are sophomores and juniors. A few seniors and an occasional first year student also take Math 171. If you are heading to graduate school in pure mathematics, you want to take Math 171 early enough so as to be able to take at least one subsequent course in abstract algebra. Also the Budapest Semesters in Mathematics requires either 171 or 131 (they prefer both and they have accepted 101 instead of 131) as an indication that you are comfortable with proofs.

What is the workload? You will be expected to actively engage the material by reading the textbook &/or watch online videos before each class. Much of the class time will be devoted to collaborative work and discussion of the material, and the assignments will be spread over the week. When students who took this class were asked how many hours outside of class they spent on Abstract Algebra, the median response was $\boxed{9}$ hours (the average was 8, the minimum was 3 and the maximum was 13). When twenty three respondents were asked "How challenging was this course compared to other courses at Pomona College?", 2 responded "average", 15 responded "above average", and 6 responded "far above average". While the class is challenging, there will be ample resources and supportive peers to make the experience a rewarding one.