(1) Find the interval of convergence of each of the following series. Be sure to check the endpoints.

(a) \(\sum_{n=1}^{\infty} \frac{(x - 1)^n}{n^2}\)  \(\sum_{n=1}^{\infty} \frac{3^{n+1}n(x + 1)^n}{\ln(n + 1)}\)

(b) \(\sum_{n=2}^{\infty} \frac{x^{n-1}}{2^n \ln(n)}\)  \(\sum_{n=0}^{\infty} n!(x - 2)^n\)

(c) \(\sum_{n=0}^{\infty} \frac{(x + 2)^{2n}}{n!}\)  \(\sum_{n=0}^{\infty} \frac{n(x + 3)^{n+1}}{n^2 + 1}\)

(2) Find the Taylor series of each of the following functions \(f(x)\) around the specified points \(a\). If you cannot find the general \(\sum\)-notation, write out the first 5 terms explicitly.

(a) \(f(x) = x^{3/2}; a = 1\)  \(f(x) = e^{-3x}; a = 1/3\)

(b) \(f(x) = \sin(2x); a = \pi/8\)  \(f(x) = e^x - e^{-x}; a = 0\)

(c) \(f(x) = \frac{1}{x^2}; a = 1\)  \(f(x) = x^3 - 3x^2 + 2x + 7; a = -1\)

(3) Find the Taylor series of each of the following functions using your favorite method. This might include the original definition, or any of the shortcuts we have come across. Specify around which \(a\) you are finding the Taylor series.

(a) \(\frac{1}{x + 3}\)  \(\frac{1}{(x + 1)^3}\)

(b) \(\ln(1 - 3x)\)  \(\cos^2(x)\)

(c) \(e^{-x+1}\)  \(\int_{0}^{x} e^t dt\)

(4) Use Taylor series methods to compute each of the following expressions.

(a) \(\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{n}\)  \(\lim_{x \to 0} \frac{\sin(x) - x + x^3/6}{x^5}\)

(b) \(\frac{3}{2} - \frac{9}{3!} + \frac{27}{4!} - \frac{81}{5!} + \cdots\)  \(\frac{\pi^2}{2^2} - \frac{\pi^4}{2^4} + \frac{\pi^6}{2^6} - \cdots\)
(5) The following questions all entail bounding the approximation error using Taylor’s remainder.

(a) How good is the approximation \( \sin(x) \approx x - x^3/6 \), if \( |x| < \pi/6 \)?

(b) Approximate \( \ln(0.9) \) using a Taylor polynomial of degree 4. What is the most that the corresponding error could be?

(c) Suppose we wish to approximate \( e^{-0.1} \) by a Taylor polynomial of degree \( n \). How big does \( n \) have to be if we wish that \( |e^{-0.1} - P_n(-0.1)| < 0.001 \)?