These are differential equations which can be written as

\[ ay'' + by' + cy = 0 \]

where \( y \) is some differentiable function of \( x \); \( a, b \) and \( c \) are constants; and \( a \neq 0 \). The set of solutions to such differentiable equations could be found as follows.

Consider the quadratic equation \( ar^2 + br + c = 0 \) and find its roots.

**Case 1.** If \( b^2 - 4ac > 0 \) then there are two real roots to the quadratic equation. Call them \( r_1 \) and \( r_2 \). The family of solutions to the differential equation is

\[ y = C_1 e^{r_1x} + C_2 e^{r_2x} \]

where \( C_1 \) and \( C_2 \) can be any numbers.

**Case 2.** If \( b^2 - 4ac = 0 \) then there is one root to the quadratic equation, namely \(-b/2a\). The family of solutions to the differential equation is

\[ y = C_1 xe^{-bx/2} + C_2 e^{-bx/2} \]

where \( C_1 \) and \( C_2 \) can be any numbers.

**Case 3.** If \( b^2 - 4ac < 0 \) then there are no real roots to the quadratic equation. Let \( \alpha = -b/2a \) and \( \beta = \sqrt{4ac - b^2}/2a \). The family of solutions to the differential equation is

\[ y = C_1 e^{\alpha x} \cos(\beta x) + C_2 e^{\alpha x} \sin(\beta x) \]

where \( C_1 \) and \( C_2 \) can be any numbers.