Homework 5

Assignment

[4] DeGroot, section 3.3 Suppose that the c.d.f. $F$ of a random variable $X$ is as given in Figure 3.9. Find each of the following probabilities:

1. $P(X = -1)$.
2. $P(X < 0)$.
3. $P(X \leq 0)$.
4. $P(X = 1)$.
5. $P(0 < X \leq 3)$.
6. $P(0 < X < 3)$.
7. $P(0 \leq X \leq 3)$.
8. $P(1 < X \leq 3)$.

Figure 3.9 The c.d.f. for Exercise 4.
9. \( P(1 \leq X \leq 2) \).
10. \( P(X > 5) \).
11. \( P(X \geq 5) \).
12. \( P(3 \leq X \leq 4) \).

[6] DeGroot, section 3.3 Suppose that the c.d.f. of a random variable \( X \) is as follows:

\[
F(x) = \begin{cases} 
  e^{x-3} & x \leq 3 \\
  1 & x > 3
\end{cases}
\]

Find and sketch the pdf of \( X \).

[8] DeGroot, section 3.3 Suppose that a point in the \( xy \)-plane is chosen at random from the interior of a circle for which the equation is \( x^2 + y^2 = 1 \); and suppose that the probability that the point will belong to each region inside the circle is proportional to the area of that region. Let \( Z \) denote a random variable representing the distance from the center of the circle to the point. Find and sketch the c.d.f. of \( Z \).

[12] DeGroot, section 3.3 For the c.d.f of Problem 6 in deGroot and Schervish, find the quantile function.

[5] DeGroot, section 3.4 Suppose that the joint pdf of two random variables \( X \) and \( Y \) is as follows:

\[
f(x) = \begin{cases} 
  c(x^2 + y) & 0 \leq y \leq 1 - x^2 \\
  0 & \text{else}
\end{cases}
\]

Determine

(a) the value of the constant \( c \)
(b) \( P(0 \leq X \leq 1/2) \)
(c) \( P(Y \leq X + 1) \)
(d) \( P(Y = X^2) \)

[7] DeGroot, section 3.4 Suppose that a point \( (X, Y) \) is to be chosen from the square \( S \) in the \( xy \)-plane containing all points \((x, y)\) such that \( 0 \leq x \leq 1 \) and \( 0 \leq y \leq 1 \). Suppose that the probability that the chosen point will be in the corner \((0,0)\) is 0.1, the probability that it will be the corner \((1,0)\) is 0.2, the probability that it will be the corner \((0,1)\) is 0.4, and the probability that it will be the corner \((1,1)\) is 0.1. Suppose also that if the chosen point is not one of the four corners of the square, then it will be an interior point of the square and will be chosen according to a constant pdf over the interior of the square. Determine

(a) \( P(X \leq 1/4) \)
(b) \( P(X + Y \leq 1) \)

[11] DeGroot, section 3.4 Consider the clinical trial of depression drugs in example 2.1.4. Suppose that a patient is selected at random from the 150 patients in that study and we record \( Y \), an indicator of the treatment group for that patient, and \( X \), an indicator of whether or not that patient relapsed. Table 3.3 contains the joint pdf of \( X \) and \( Y \).

a. Calculate the probability that a patient selected at random from this study used Lithium (either alone or in combination with Imipramine) and did not relapse.
b. Calculate the probability that the patient had a relapse (without regard to the treatment group).

[R1] **Estimate** \( \pi \). Estimate the area of the circle of radius 1/2 with center at (1/2, 1/2) inside the unit square by choosing 1000 points at random. Compare your results with the true value of the area of the circle, and use your results to estimate the value of \( \pi \). How accurate is your estimate?

**Sample R example** Let’s say that I want to estimate the area under \( y = \frac{1}{x+1} \) in the unit square. I simulate random point, test whether it is under the curve, and keep the points that are within the area constraints.

```r
nreps = 20
xval = runif(nreps,0,1)
yval = runif(nreps,0,1)
xval  # remove when you understand runif, also try ?runif

## [1] 0.976223859 0.643624632 0.928764787 0.772491334 0.731950120
## [6] 0.001964607 0.119060708 0.896066576 0.175537640 0.160958866
## [11] 0.406134192 0.84599073 0.656781589 0.558213093 0.959637635
## [16] 0.901704642 0.051065459 0.273421782 0.995845226 0.059829889

yval  # remove when you understand runif, also try ?runif

## [1] 0.18226907 0.23756290 0.70724837 0.05527410 0.63369822 0.04342274
## [7] 0.73199782 0.81663758 0.81663436 0.83860965 0.06577875 0.81087966
## [13] 0.34716515 0.31297853 0.12167092 0.12040137 0.10861418 0.25123985
## [19] 0.53552634 0.65552565

yval < 1/(xval+1)  # remove after you understand this line

## [1] TRUE TRUE FALSE TRUE FALSE TRUE TRUE TRUE TRUE
## [12] FALSE TRUE TRUE TRUE TRUE TRUE TRUE FALSE TRUE

sum(yval < 1/(xval+1))

## [1] 15

sum(yval < 1/(xval+1))/nreps

## [1] 0.75

# by doing the integration, I see that my answer should be close to ln(2)
# up the reps (and remove the unneeded lines above!) to see if my method is close
log(2)

## [1] 0.6931472