Homework 9

Assignment

[17] DeGroot, section 3.11
Suppose that $X$ and $Y$ are random variables. The marginal p.d.f. of $X$ is

$$f(x) = \begin{cases} 3x^2 & \text{for } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Also, the conditional p.d.f. of $Y$ given that $X = x$ is

$$g(y|x) = \begin{cases} 3y^2 & \text{for } 0 < y < x \\ 0 & \text{otherwise.} \end{cases}$$

Determine

a. the marginal p.d.f. of $Y$

b. the conditional p.d.f. of $X$ given that $Y = y$.

[6 & 7] DeGroot, section 3.10
Suppose that a student will be either on time or late for a particular class and that the events that she is on time or late for the class on successive days form a Markov chain with stationary transition probabilities.

Suppose also that if she is late on a given day, then the probability that she will be on time the next day is 0.8. Furthermore, if she is on time on a given day, then the probability that she will be late the next day is 0.5.

6a. If the student is late on a certain day, what is the probability that she will be on time on each of the next three days?

6b. If the student is on time on a given day, what is the probability that she will be late on each of the next three days?

7a. If the student is late on the first day of class, what is the probability that she will be on time on the 4th day of class?

7b. If the student is on time on the first day of class, what is the probability that she will be on time on the fourth day of class?

Look at how fun it is to multiply matrices in R!!! Notice that the matrix below can’t possible be a transition matrix because the numbers are all bigger than 1 so are not probabilities.

```r
sampleP = matrix(c(1,2,3,4,5,6,7,8,9), ncol=3, byrow=TRUE)
```

```r
sampleP
```
```r
sampleP2 = sampleP %*% sampleP
sampleP2

sampleP2 %*% sampleP  # 3 steps

sampleP2 %*% sampleP2  # 4 steps
```

[12] DeGroot, section 3.10
Suppose that three boys A, B, C are throwing a ball from one to another. Whenever A has the ball, he throws it to B with a probability of 0.2 and to C with a probability of 0.8. Whenever B has the ball, he throws it to A with probability of 0.6 and to C with a probability of 0.4. Whenever C has the ball, he is equally likely to throw it to either A or B.

a. Consider this process to be a Markov chain and construct the transition matrix.

b. If each of the three boys is equally likely to have the ball at a certain time $n$, which boy is most likely to have the ball at time $n + 2$?

Suppose that one word is to be selected at random from the sentence:
THE GIRL PUT ON HER BEAUTIFUL RED HAT.
If $X$ denotes the number of letters in the word that is selected, what is the value of $E[X]$?

[6] DeGroot, section 4.1 Suppose that a random variable $X$ has a continuous distribution with the p.d.f.:

$$f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the expectation of $1/X$.  

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Suppose that a random variable $X$ has the uniform distribution on the interval $[0, 1]$. Show that the expectation of $1/X$ is infinite.

(R1) Write code to take (how many?) steps in the following Markov chain. Plot the percentage of time spent in states a, b, and c. How many steps did your Markov Chain need to take before the probabilities were stable?

```r
sampleP = matrix(c(0.8,.2,.65,.35), ncol=2, byrow=TRUE)
convP = diag(2)
convP

## [,1] [,2]
## [1,] 1 0
## [2,] 0 1

steps=10

for (i in 1:steps){
  convP = convP %*% sampleP
}

convP

## [,1] [,2]
## [1,] 1 1
## [2,] 1 1
```
```r
## [1,] 0.7647059 0.2352941
## [2,] 0.7647059 0.2352941

barplot(convP[1,], names.arg=c("a", "b"), ylim=c(0,1), main="State Probabilities")
```