Random Variables & Distributions

- **random variable** Let $S$ be the sample space for an experiment. A real-valued function that is defined on $S$ is called a random variable.

- **distribution** Let $X$ be a random variable. The distribution of $X$ is the collection of all probabilities of the form $P(X \in C)$ for all sets $C$ of real numbers such that $\{X \in C\}$ is an event.

- **discrete** A random variable $X$ is discrete if it can take only a finite number $k$ of different values, $x_1, x_2, \ldots, x_k$ or, at most, an infinite sequence of different values $x_1, x_2, \ldots$.

- **probability function** If a random variable $X$ has a discrete distribution, the probability function of $X$ is defined as the function $f$ such that for every real number $x$,

  $$f(x) = P(X = x)$$

- **Bernoulli** A random variable $Z$ that takes only two values 0 and 1 with $P(Z = 1) = p$ has a Bernoulli distribution with parameter $p$.

- **uniform** Let $a \leq b$ be integers. Suppose that the value of a random variable $X$ is equally likely to be each of the integers $a, \ldots, b$. Then we say that $X$ has a uniform distribution on the integers $[a, b]$.

- **binomial** A binomial random variable is a sum of Bernoulli random variables.

- **continuous** A random variable $X$ is continuous if there exists a nonnegative function $f$, defined on the real line, such that for every interval of real numbers, the probability that $X$ takes a value in the interval is the integral of $f$ over the interval. For example:

  $$P(a \leq X \leq b) = \int_a^b f(x)dx$$

- **probability density function** If $X$ has a continuous distribution, the function $f$ described above is called the probability density function (pdf).

- **cumulative distribution function** The cumulative distribution function (cdf) of a random variable is the function:

  $$F(x) = P(X \leq x)$$