Assignment #1

Due on Friday, September 13, 2019

Read Section 2.1 on the *Definition of Euclidean Space* in the class Lecture Notes.

Read Section 2.2 on *Spans, Lines and Planes* in the class Lecture Notes.

Do the following problems

1. Let $v_1 = \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix}$.

   (a) Give the parametric equations of the line through the point $P\colon (0, 4, 7)$ in the direction of the vector v_1.

   (b) Give the equation of the plane through the point $P(0, 4, 7)$ spanned by the vectors v_1 and v_2.

2. The following give parametric equations of two lines in $\mathbb{R}^3$:

$$\left\{\begin{array}{lcl} x & = & -1 + 4t \\ y & = & -7t \\ z & = & 2 - t \end{array}\right. \qquad \left\{\begin{array}{lcl} x & = & -1 + s \\ y & = & 2 - s \\ z & = & 2s \end{array}\right.$$

   Determine if the two lines ever meet. Justify your answer. If the lines do meet, give the equation of the plane that contains both lines.

3. The following give parametric equations of two lines in $\mathbb{R}^3$:

$$\left\{\begin{array}{lcl} x & = & 2 + 4t \\ y & = & -1 - 7t \\ z & = & 2 - t \end{array}\right. \qquad \left\{\begin{array}{lcl} x & = & s \\ y & = & 1 - s \\ z & = & -2 + 2s \end{array}\right.$$

   Determine if the two lines ever meet. Justify your answer. If the lines do meet, give the equation of the plane that contains both lines.

4. The vectors $v_1 = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $v_3 = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$ in $\mathbb{R}^3$ can span a line, a plane or the entire three dimensional Euclidean space $\mathbb{R}^3$. Give the equation of the geometric object that the vectors v_1, v_2 and v_3 span. Assume the vectors v_1, v_2 and v_3 are in standard position.

5. Consider the plane in $\mathbb{R}^3$ whose equation is

$$x - 4y + 7z = 3. \tag{1}$$

   (a) Find two vectors, v_1 and v_2, that span the plane given by the equation in (1).

   (b) Express the plane given by the equation in (1) in the form

$$\overrightarrow{OP} + \operatorname{span}(\{v_1, v_2\}),$$

   where P is a point in the plane.