Assignment #12

Due on Monday, March 30, 2020

Read Section 5.3 on *Moments* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Section 4.4 on *Moments* in DeGroot and Schervish.

Do the following problems

1. Suppose that X is a random variable for which the mgf is as follows:
$$\psi_X(t) = e^{t^2+3t} \quad \text{for } -\infty < t < \infty.$$
Find the mean and variance of X.

2. Suppose that X is a random variable for which the mgf is as follows:
$$\psi_X(t) = \frac{1}{6}(4 + e^t + e^{-t}) \quad \text{for } -\infty < t < \infty.$$
Find the probability distribution of X.

3. Let X be a random variable with moment generating function (mgf) ψ_X.

   (a) Let $Y = cX$, where c is a constant. Compute the mgf of Y in terms of ψ_X.
   (b) Let $Y = X + a$, where a is a constant. Compute the mgf of Y in terms of ψ_X.
   (c) Put $Y = \dfrac{X - \mu}{\sigma}$. Compute the mgf of Y in terms of ψ_X. Use this mgf to compute $E(Y)$ and $\text{Var}(Y)$.

4. Let X have pdf given by
$$f_X(x) = \begin{cases} \dfrac{1}{2}x^2 e^{-x}, & \text{if } x > 0; \\[2mm] 0, & \text{if } x \leqslant 0. \end{cases}$$
Compute the mgf of X and use it compute $E(X)$, $E(X^2)$ and $\text{Var}(X)$.

5. Let X have mgf
$$\psi_X(t) = \frac{1}{6}e^{-2t} + \frac{1}{3}e^{-t} + \frac{1}{2}e^t, \quad \text{for all } t \in \mathbb{R}.$$
Compute $\Pr(|X| \leqslant 1)$.