

Assignment #9

Due on Monday, March 9, 2020

Read Section 5.1 on *Expected Value of a Random Variable* in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

Read Section 4.1, *The Expectation of a Random Variable*, in DeGroot and Schervish.

Do the following problems

1. Let $X \sim \text{Uniform}(a, b)$. Compute $E(X)$.
2. Let X be a continuous random variable with pdf

$$f_X(x) = \frac{1}{\pi(x^2 + 1)} \text{ where } x \in \mathbb{R}.$$

Show that X has no expectation.

3. Suppose that X is a **bounded** and continuous random variable; that is, there exists a positive number M such that

$$\Pr(|X| \leq M) = 1.$$

Show that $E(X)$ exists. In other words, show that

$$\int_{-\infty}^{\infty} |x|f_X(x) \, dx < \infty.$$

4. Suppose a random variable X has a uniform distribution on the interval $[0, 1]$. Show that the expectation of $1/X$ does not exist.
5. Suppose that a point is chosen at random on a stick of unit length at that the stick is broken into two pieces at that point. Find the expected value of the length of the longer piece.