Given a chemical reaction network, we can describe it in terms of species, reactions, and complexes. For the network below, our species are $A$, $B$, $C$ while the complexes are $A + C$, $B + C$, and $2B$.

\[
A + C \quad B + C \quad 2B
\]

Edges representing reactions are drawn as arrows, with their reaction-rate values and $\kappa$-terms per reaction in the network, and the monomials are composed of the species in the network. They have one term per reaction in the network, and the monomials are composed by the reaction complex. Here, $f_A$ has coefficient $2\kappa_1$ since it has a net change of 2 across the reaction. Thus,

\[
\frac{dx_A}{dt} = f_A = 2\kappa_1 x_B + \kappa_2 x_B
\]

\[
\frac{dx_B}{dt} = f_B = -2\kappa_1 x_B - \kappa_2 x_B
\]

We find the steady-state variety by calculating the solutions to $f_A, f_B = 0$. In this case, it is $x_B = 0, x_A = \frac{\kappa_2}{2\kappa_1}$.

### Definitions

**Given a chemical reaction network**, $G$, we can describe it in terms of species, reactions, and complexes. For the network below, our species are $A$, $B$, $C$ while the complexes are $A + C$, $B + C$, and $2B$.

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### Positive Steady-State Variety

**The two reactant complexes have the same concentration such that their sum is zero for all species $i$, $\kappa_i = 0$ for all species $i$, $x_i$ is at-most-bimolecular, and only if the following criteria are true:**

- One reactant complex is $A + B$ and the other is monomolecular.
- The columns of the stoichiometric matrix are negative multiples of each other.
- The supports of the reactant complexes are nonempty and distinct (not necessarily disjoint).

### Future Work

- Investigating and classifying other types of chemical reaction networks, especially the following:
  - at-most-bimolecular networks
  - 3-species, 2-reaction networks
  - $n$-species and higher networks where $n$ is at least 4
- Considering other applications of these techniques, such as complicated biochemical reactions.

### References

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