Adinkras as Origami

Arsh Chhabra 1 Xuehuai He 1 Elena O’Grady 2 Melinda Yang 1 Cameron Thomas 3 Edray Goins 3

1Pomona College 2Reed College 3University of Georgia

Abstract

Around 20 years ago, physicists Michael Farak and Jim Gates invented Adinkras as a way to better understand Supersymmetry. These are biparti-
tite graphs whose vertices represent bosons and fermions, and whose edges represent operators which relate the particles. Recently, Doran et al. 2023 determined that Adinkras are a type of Bruhat (Bruhat) by explicitly exhibiting a Belyĭ map as a composition \( f: S \to P^2(\mathbb{C}) \). We are interested in exhibiting the same Belyĭ map as a composition \( f: S \to E(\mathbb{C}) \).

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Adinkras

Let \( F_2 = \{ 0, 1 \} \) be the finite field of 2 elements. Fix an integer \( n \geq 2 \). Denote \( \mathbb{F}_2^n \) as the n-dimensional vector space over this field, where a vector \( v = (v_1, v_2, ..., v_n) \) has components \( v_i \in \mathbb{F}_2 \).

An Adinkra is a bipartite graph constructed as follows. Define \( \mathbb{F}_2^n \to \mathbb{F}_2 \) via counting the number of non-zero components \( v_i \). Choose a subspace \( C \subset \mathbb{F}_2^n \) and \( \beta \in \mathbb{F}_2^n \). We form an Adinkra from the elliptic curve \( \mathbb{E}(C) \), which has dimension \( n = 0 \). We form an Adinkra from the elliptic curve \( E : y^2 = x^3 - x \).

Examples of Adinkras as Belyĭ maps

Consider \( n = 4 \) and the subspace \( C = \{ 0000 \} \), which has dimension \( n = 0 \).

For any positive integer \( n \), consider the map \( \beta: \mathbb{F}_2^n \to \mathbb{F}_2 \) given by \( \beta(1) = 1 \) and \( \beta(0) = 0 \).

This is a \( \beta \)-Adinkra of degree \( n \).

The corresponding Doran et al. 2023 has one “black” vertex \( B = \{ 0 \} \), one “white” vertex \( W = \{ 0 \} \), which \( \beta \)-edges, and \( |\beta| = n \) faces

Belyĭ Maps and Dessins d’Enfants

Every compact, connected Riemann surface \( S \) is a smooth curve, that is, can be defined by a single polynomial \( f(x) \). A Belyĭ map is a natural function \( \beta: S \to P^2(\mathbb{C}) \) which satisfies \( \beta(\mathbb{E}(C)) = 1 \).

A Dessin d’Enfant is a bipartite graph on \( S \) corresponding to the projimage of \( \{ 0, 1 \} \) on \( P^2(\mathbb{C}) \) under a Belyĭ map \( \beta: S \to P^2(\mathbb{C}) \).

Adinkras as Dessins d’Enfant

In Doran et al. 2023 proved the following. For an integer \( n \geq 2 \), fix a primitive congruent 2nd root of unity \( \xi \). Let \( s \to \mathbb{E}(C) \to \mathbb{P}^2(\mathbb{C}) \) be a Belyĭ map that identifies \( \beta = 0 \) and \( \beta = 1 \).

We may tile \( S \) by \( \mathbb{E}(C) \) and \( \xi \) is a root of unity of order \( 2n \). Let \( \xi = \sqrt{-1} \).

Examples of Belyĭ Maps as Adinkras

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References


