Math 151, Spring 2011
Jo Hardin, HW #6

Homework due on THURSDAY, FEBRUARY, 24TH, START OF CLASS.

1. DeGroot (3^{rd} or 4^{th} ed.), section 3.3: # 5, 11

2. DeGroot (3^{rd} or 4^{th} ed.), section 3.4: # 5, 7, 8

3. DeGroot (3^{rd} or 4^{th} ed.), section 3.5: # 4, 11

4. DeGroot # 15, Section 3.5: A painting process consists of two stages. In the first stage, the paint is applied, and in the second stage, a protective coat is added. Let X be the time spent on the first stage, and let Y be the time spent on the second stage. The first stage involves an inspection. If the paint fails the inspection, one must wait three minutes and apply the paint again. After a second application, there is no further inspection. The joint p.d.f of X and Y is:

$$f(x,y) = \begin{cases} 1/3 & 1 < x < 3 \text{ and } 0 < y < 1 \\ 1/6 & 6 < x < 8 \text{ and } 0 < y < 1 \\ 0 & \text{else} \end{cases}$$

   (a) Sketch the region where $f(x,y) > 0$. Note that it is not exactly a rectangle.

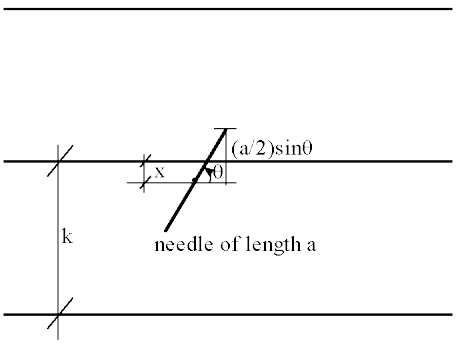
   (b) Find the marginal p.d.f.s of X and Y.

   (c) Show that X and Y are independent.

5. Additional problem in R: A needle of length a is dropped randomly on a plane ruled with parallel lines that are a distance k apart ($k \geq a$).

   Show that:

$$P(\text{needle crosses a line}) = \frac{2a}{\pi k}$$



   Let θ be the angle in which the needle falls ($\theta \in [0, \pi/2]$). Let x be the distance from the needle center to the closest line ($X \in [0, \frac{k}{2}]$). The needle will cross the line if $X < \frac{a}{2}\sin(\theta)$.

   (a) ANALYTICALLY (which means an exact answer using integrals.) [Hint: θ and X are independent uniform random variables. First write down the marginal pdfs of θ and X. Their joint distribution is simply their product.]

(b) COMPUTATIONALLY [First, pick values for k and a ($k > a$). In R, let's say I want to simulate two independent uniform random variables and measure how often the first is less than the second squared:

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?runif
X = runif(10000, 0, 1)
Y = runif(10000, -2, 2)
sum(X < Y^2)          # the number of times X < Y^2
sum(X < Y^2) / 10000  # the proportion of times X < Y^2
```

or if I wanted to do some trig:

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sin(pi)
sin(pi/2)
sin(pi/4)
sin(2*pi)
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(c) Given what you calculated above in (a) and (b), describe a procedure for approximating π. Using the result from part (b), give an estimate for the value of π.