Math 151, Spring 2011
Jo Hardin, HW #8

Homework due on THURSDAY, MARCH 10^{th}, START OF CLASS.

1. DeGroot (3^{rd} or 4^{th} ed.), section 3.7: # 4, 7, 8

2. DeGroot (3^{rd} or 4^{th} ed.), section 3.8: # 2, 8, 9, 11, 14

3. DeGroot # 17, Section 3.8: An insurance agent sells a policy which has a \$100 deductible and a \$5000 cap. This means that when the policy holder files a claim, the policy holder must pay the first \$100. After the first \$100, the insurance company pays the rest of the claim up to a maximum payment of \$5000. Any excess must be paid by the policy holder. Suppose that the dollar amount X of a claim has a continuous distribution with pdf $f(x) = 1/(1+x)^2$ for $x > 0$ and 0 otherwise. Let Y be the amount that the insurance company has to pay on the claim.

   (a) Write Y as a function of X, i.e., $Y = r(X)$.

   (b) Find the cdf of Y.

   (c) Explain why Y has neither a continuous nor a discrete distribution.

4. DOES A QUADRATIC FUNCTION WITH RANDOM COEFFICIENTS HAVE REAL ROOTS? Let A, B, and C be independent random variables uniform on [0, 1]. What is the probability that the roots of the quadratic $A * x^2 + B * x + C$ are real? (Source: Rice *Mathematical Statistics and Data Analysis third edition* exercise 3.11).

   (a) In R, simulate the above situation. If you don't know when the root of an equation will be real, look up how to solve for the roots of a quadratic function. According to your simulation, how often are the roots real? [As always, if you want help with R, tell me exactly what you want to do, and I'll tell you how to do it in R.]

   (b) Analytically: find the probability that three random coefficients will produce an equation with real roots. [Hint: Let $Y = B^2$, $W = 4AC$. Find the distributions of each of Y and W. Note that Y and W are independent. Because we haven't yet covered section 3.9, I will tell you the distribution of W.]
   The distribution of W is given by:

$$f(w) = \begin{cases} -\log(w/4)/4 & \text{if } 0 \le w \le 1 \\ 0 & \text{otherwise} \end{cases}$$