Clinical Trial Example

In the clinical trial, suppose that Prob(no relapse) $= p$, but we don't know p (among other things, it depends on the treatment). Partition S into B_j s.t. $\frac{j-1}{10} = p$ in $B_j, j = 1, \ldots 11$. For example, take the first person in group 1, *e.g. iprimamine group*, let $E_1 =$ event person 1 has no relapse. Suppose: $P(E_1|B_j) = \frac{j-1}{10}$.

If we have *no* information about p, then all B_j are equally likely. Find $P(E_1)$.

b. $\frac{1}{2}$

Guess first!
Since all B_j are equally likely, $P(B_j) = \frac{1}{11}$, for all j.

$$P(E_1) = \sum_{j=1}^{11} P(E_1|B_j)P(B_j) = \sum_{j=1}^{11} \frac{1}{11}\frac{j-1}{10} = \frac{55}{110} = \frac{1}{2}$$

In the clinical trial example, we observe that the first patient did *not* have a relapse. Recall that we assumed that, if all B_j were equally likely, $P(E_1) = .5$. How does the data change our belief? I.e., how does it change the probabilities of the occurrence of each of the B_j?

What is the notation for the posterior (given the first person did not relapse) probability of the j^{th} value for the probability of relapse?

e. $P(B_j|E_1)$

Note that, for any two sets, $P(E|F) = \frac{P(EF)}{P(F)} \Rightarrow P(EF) = P(E|F)P(F)$. We've used this before. The proof of Bayes' Theorem involves writing out the definition of conditional probability, and using this fact:

$$P(B_j|A) = \frac{P(B_j \cap A)}{P(A)} = \frac{P(A|B_j)P(B_j)}{\sum_j P(A|B_j)P(B_j)}.$$

In the example:

$$P(B_j|E_1) = \frac{P(E_1|B_j)P(B_j)}{.5} = \frac{j-1}{55}$$

The value $\frac{1}{11}$ is the *prior* for the value of $P(B_j)$.
The value $\frac{j-1}{55}$ is the *posterior* for the values of $P(B_j)$.

If we use the results from all 40 of the patients in the imipramine group:

The point here is to have some sense of the probability of no relapse given the data we observed. We don't really believe that all of the B_j are equally likely (that probability of $p = 0$ is just the same as $p = 0.5$). But at the same time, we don't presume to *know* that $p = 22/40 = 0.55$. The value of 0.55 is a statistic (comes from the data) and is an estimate of the parameter (number from the population) not the **truth** of the population. So again we ask, How do the data change our belief? I.e., how do they change the probabilities of the occurrence of each of the B_j?

	Treatment Group				
Response	*Imipramine*	*Lithium*	*Comb.*	*Placebo*	**Total**
Relapse	18	13	22	24	77
No relapse	22	25	16	10	73
Total	40	38	38	34	150

What is the probability of the data (22 successes, 18 failures) given a particular B_j?

e. $\binom{40}{22} \left(\frac{j-1}{10}\right)^{22} \left(\frac{11-j}{10}\right)^{18}$

Let A be the event "22 successes and 18 failures". Then we know:

$$P(A|B_j) = \binom{40}{22} \left(\frac{j-1}{10}\right)^{22} \left(\frac{11-j}{10}\right)^{18}.$$

Recall that our *prior* on the $P(B_j)$ is that they are all equally likely.

Bayes' Theorem $\Rightarrow$

$$P(B_j|A) = \frac{\frac{1}{11}\binom{40}{22}\left(\frac{j-1}{10}\right)^{22}\left(\frac{11-j}{10}\right)^{18}}{\sum_{j=1}^{11} \frac{1}{11}\binom{40}{22}\left(\frac{j-1}{10}\right)^{22}\left(\frac{11-i}{10}\right)^{18}}$$

Posterior Probabilities: $P(B_j)$